
Real-time ionospheric data processing and modeling: an empirical orthogonal function approach

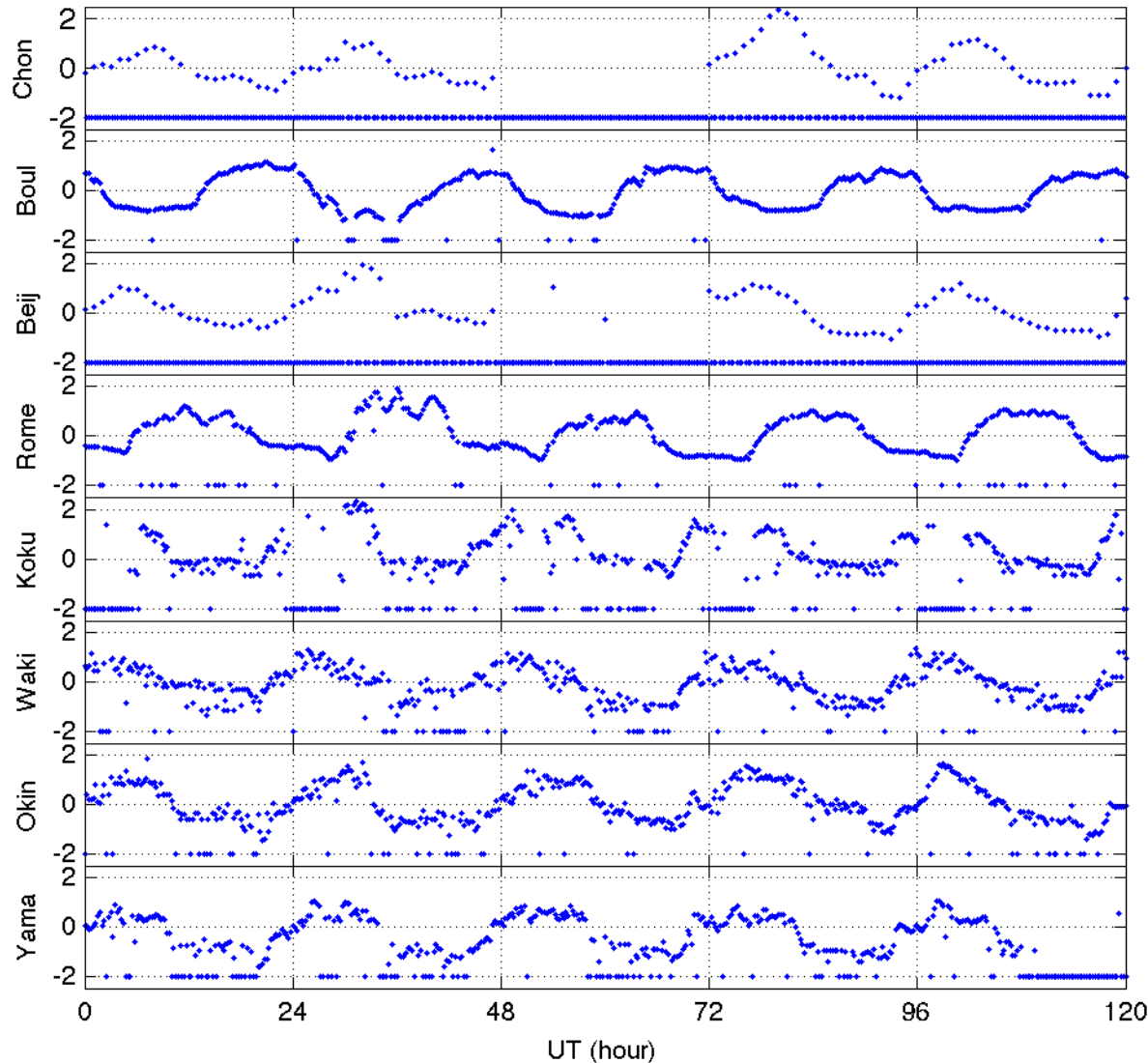
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MIT Haystack Observatory

with contributions from A. Coster and Z. Chen

Outline

- Motivation
- EOF concept
- EOF-based North America GPS-TEC modeling
- EOF-based Near Real-time data processing
 - DIEOF procedure
 - Case studies
- Summary

Data gaps and non-uniform data resolution



Test data of
normalized foF2
from WDC during
October 2013.

Empirical Orthogonal Function (EOF)

$$T(t, n, \phi, \lambda) = \sum_{i=1}^I b_i(\phi, \lambda) P_i(t, n)$$

- A decomposition of a data set in terms of orthogonal basis functions which are determined from the data.
- The i th basis function is chosen to be orthogonal to the basis functions from the first through $i - 1$, and to minimize the residual variance. (The basis functions are chosen to be different from each other, and to account for as much variance as possible.)

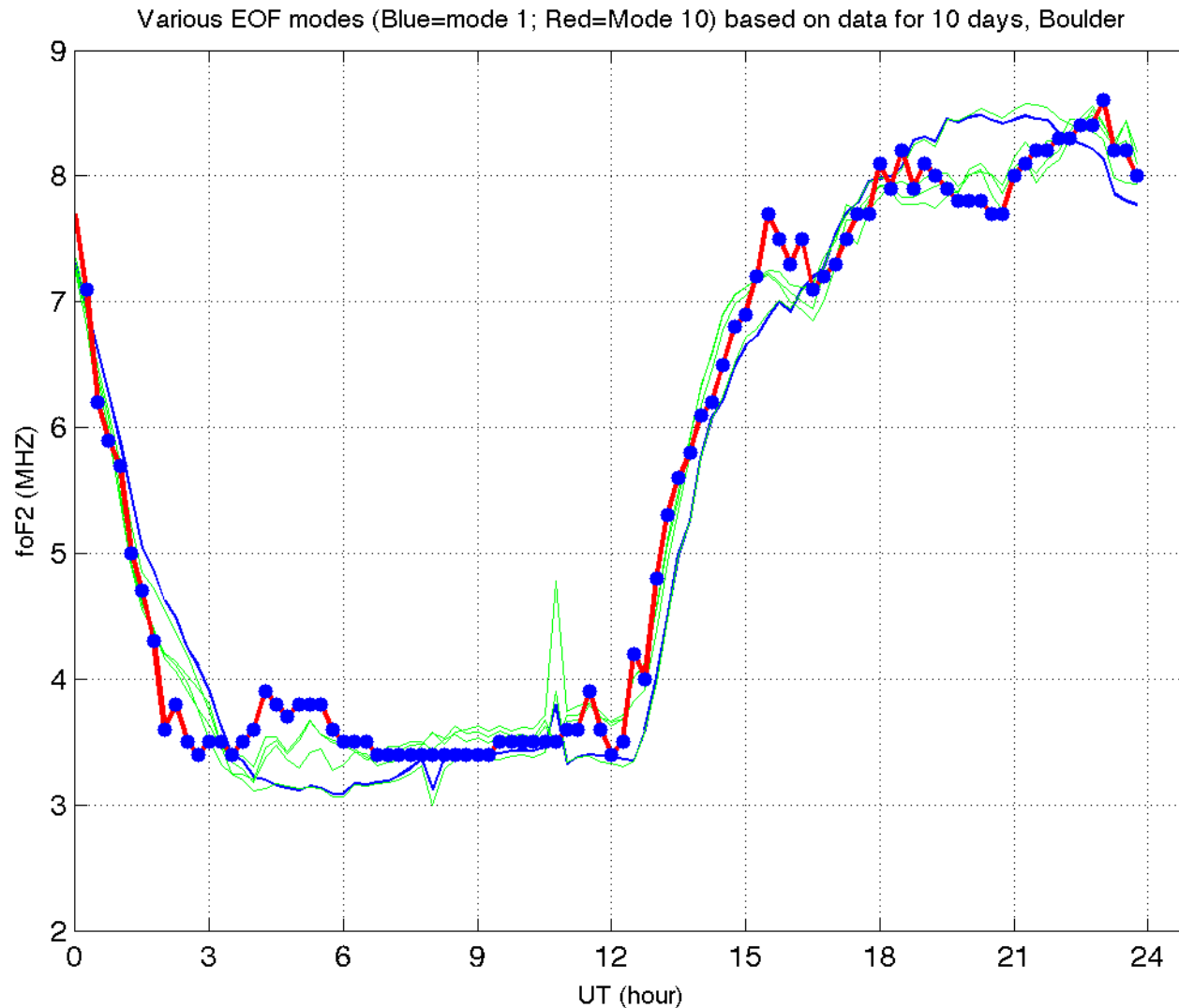
Principal Component Analysis (PCA)

$$\mathbf{C} \mathbf{b}_i = \omega_i \mathbf{b}_i$$
$$\mathbf{C} = \text{cov} [T(t, n, \phi, \lambda)]$$

Singular Value Decomposition (SVD)

$$\mathbf{T} = \mathbf{U} \mathbf{\Sigma} \mathbf{W}^T$$

EOF modes



A test dataset for 10 days over Boulder, CO was processed with EOF decomposition.

Faithfully reproduce the original data with a small number of modes

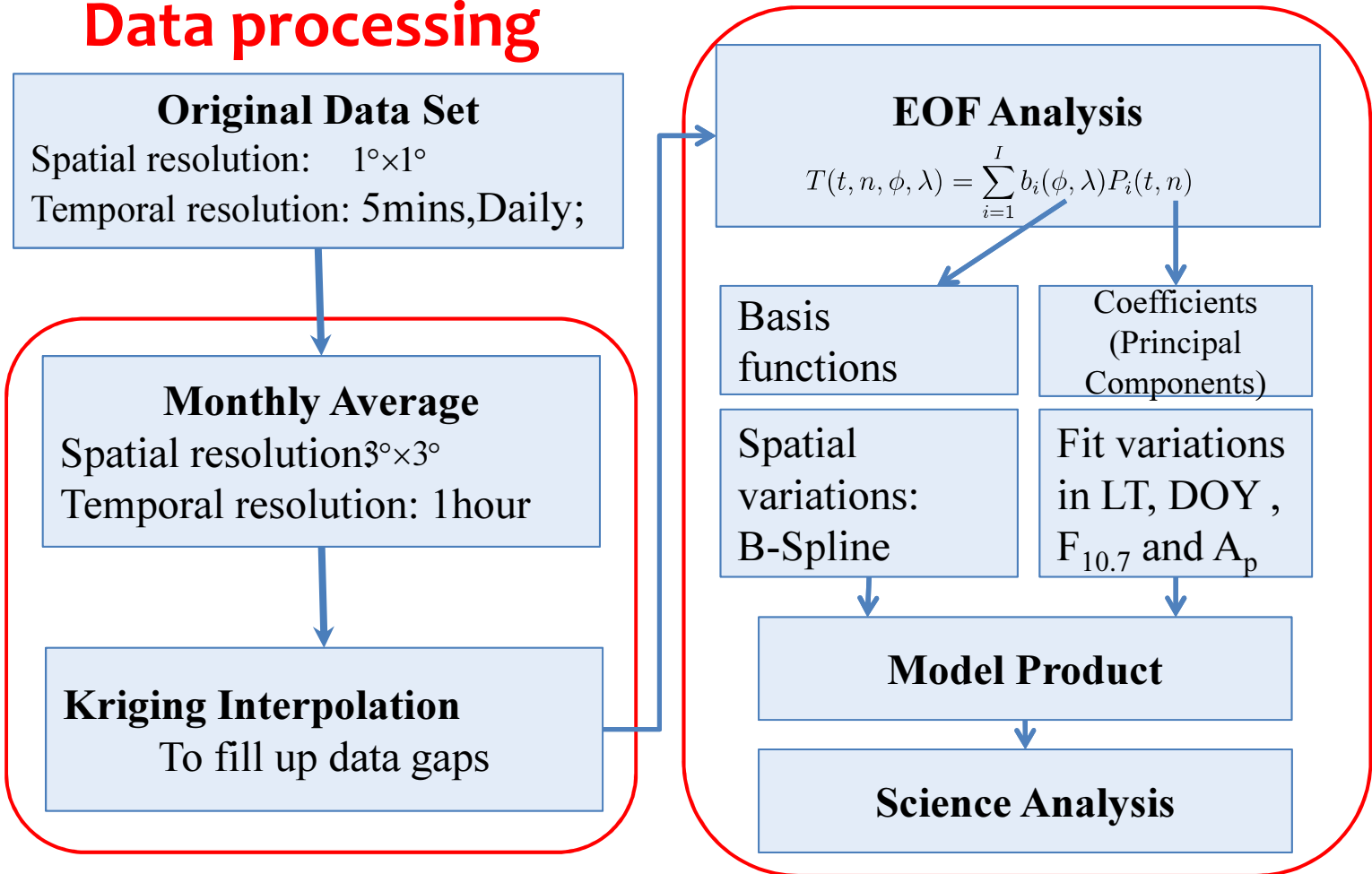
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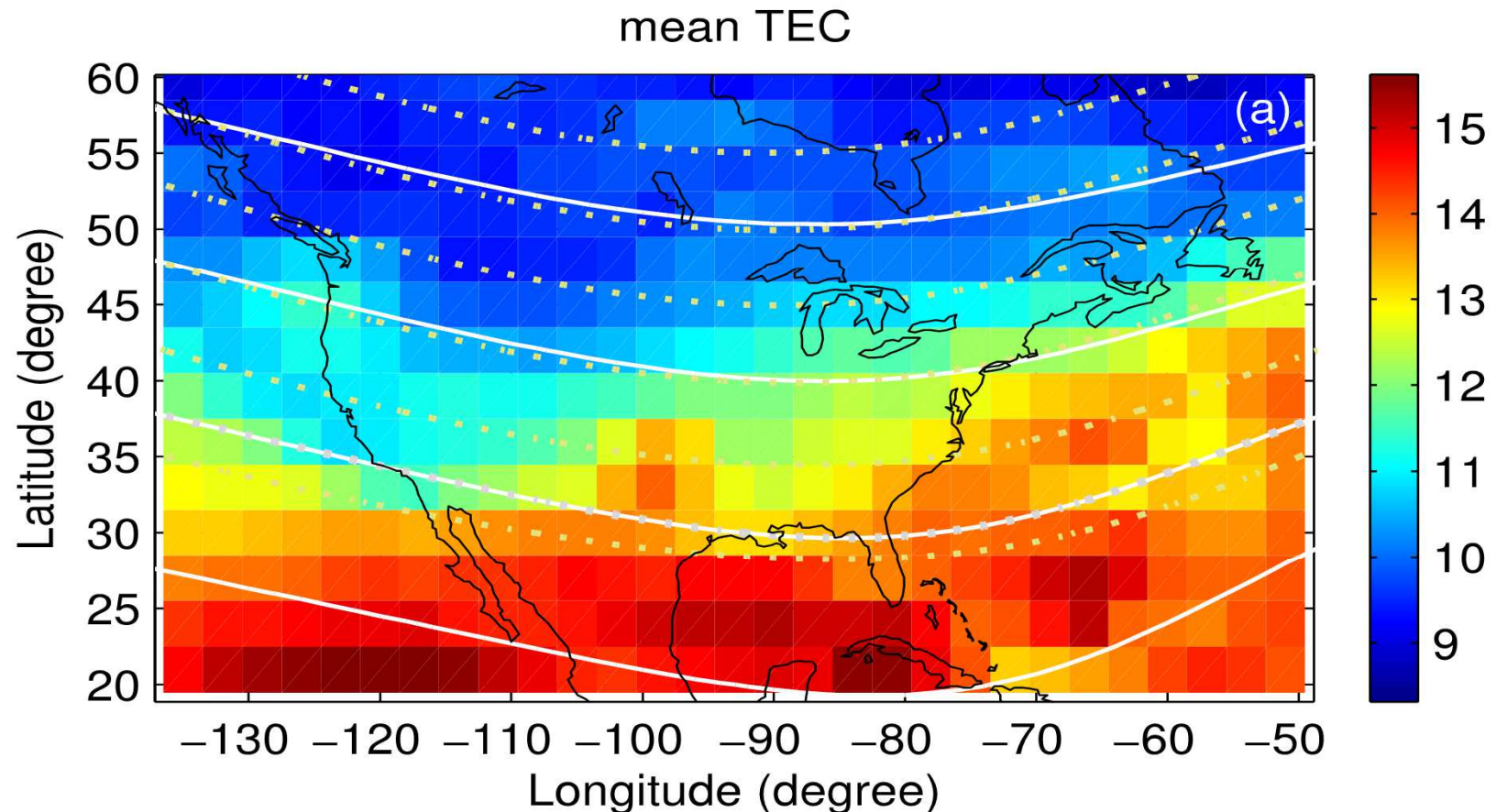
North America GPS TEC Model

based on GPS TEC data for a full solar cycle 2000 – 2012

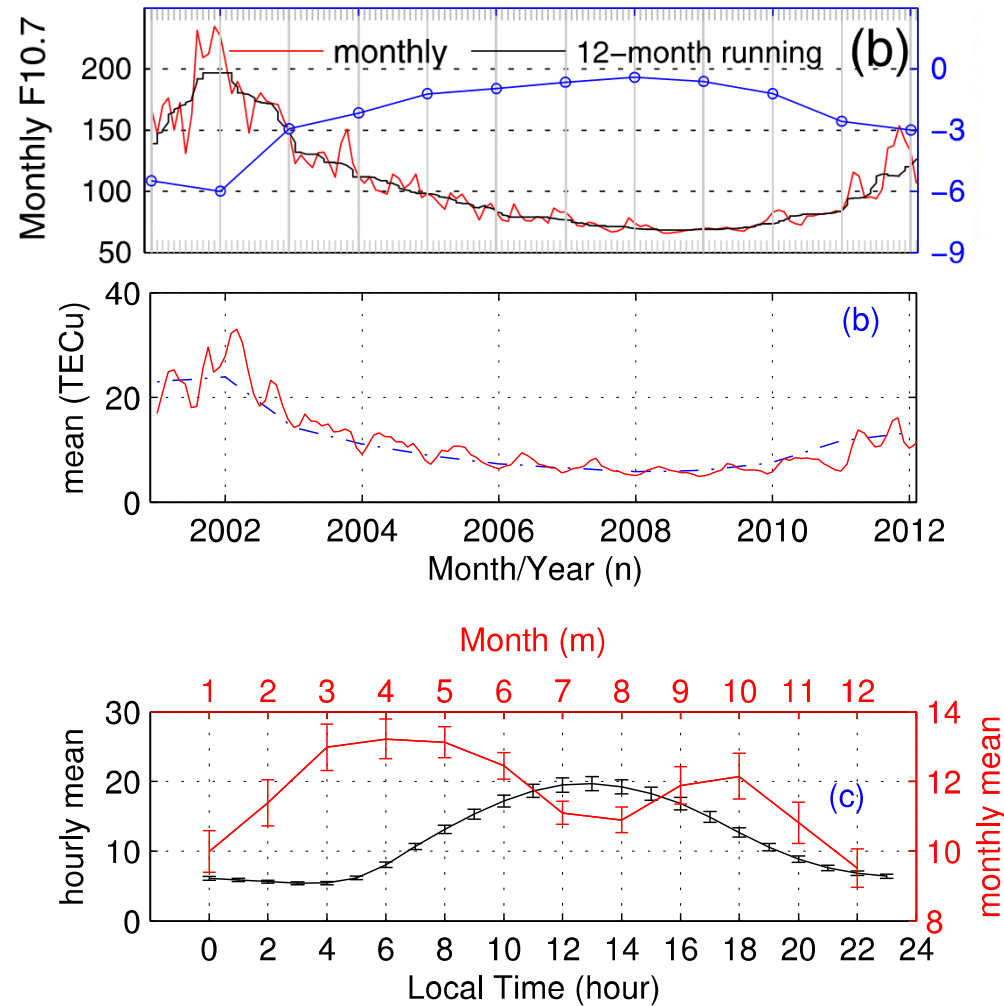
Data processing



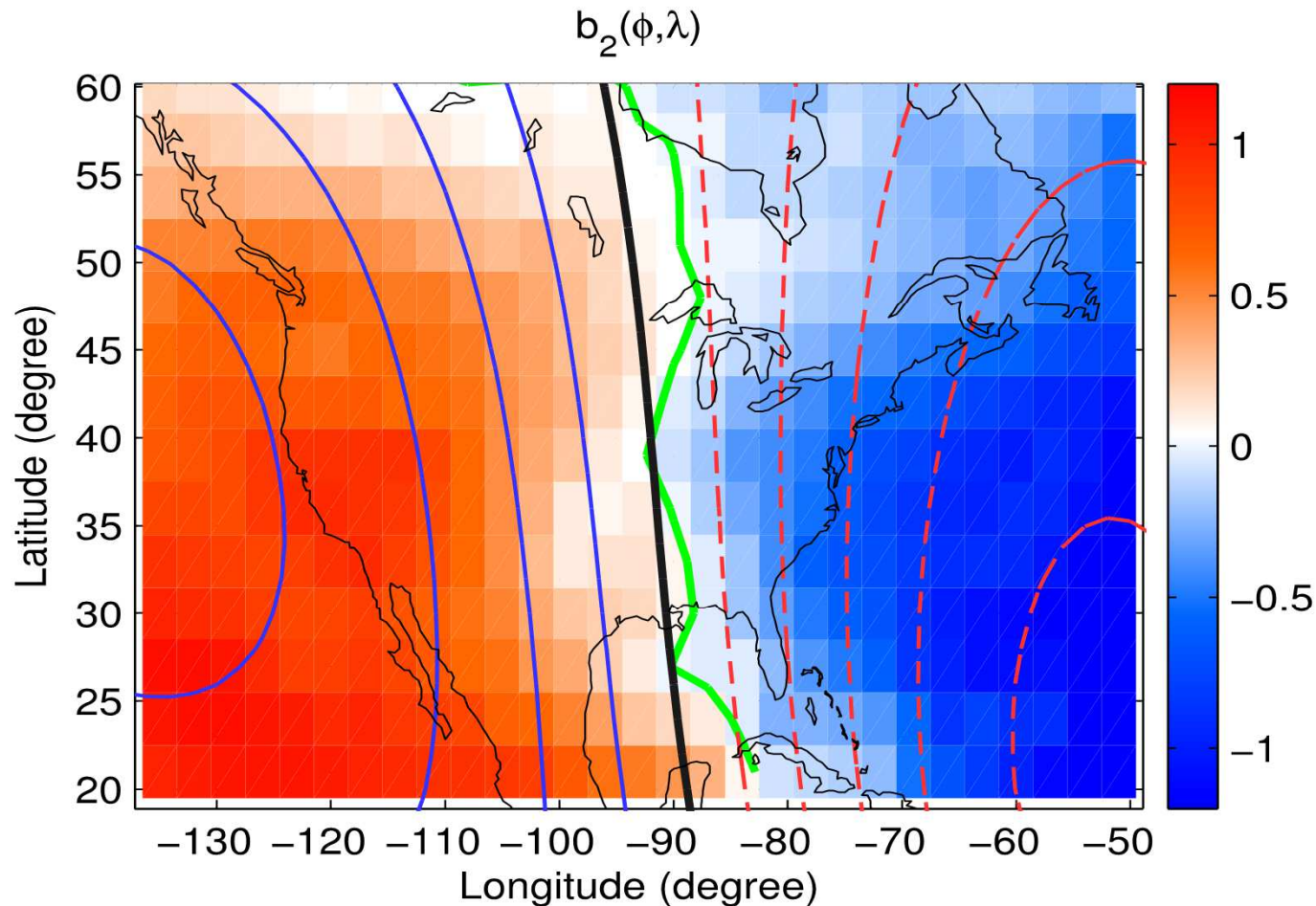
Mean TEC (spatial variations from EOF basis function mode 1)



Mean temporal variation (EOF mode 1)

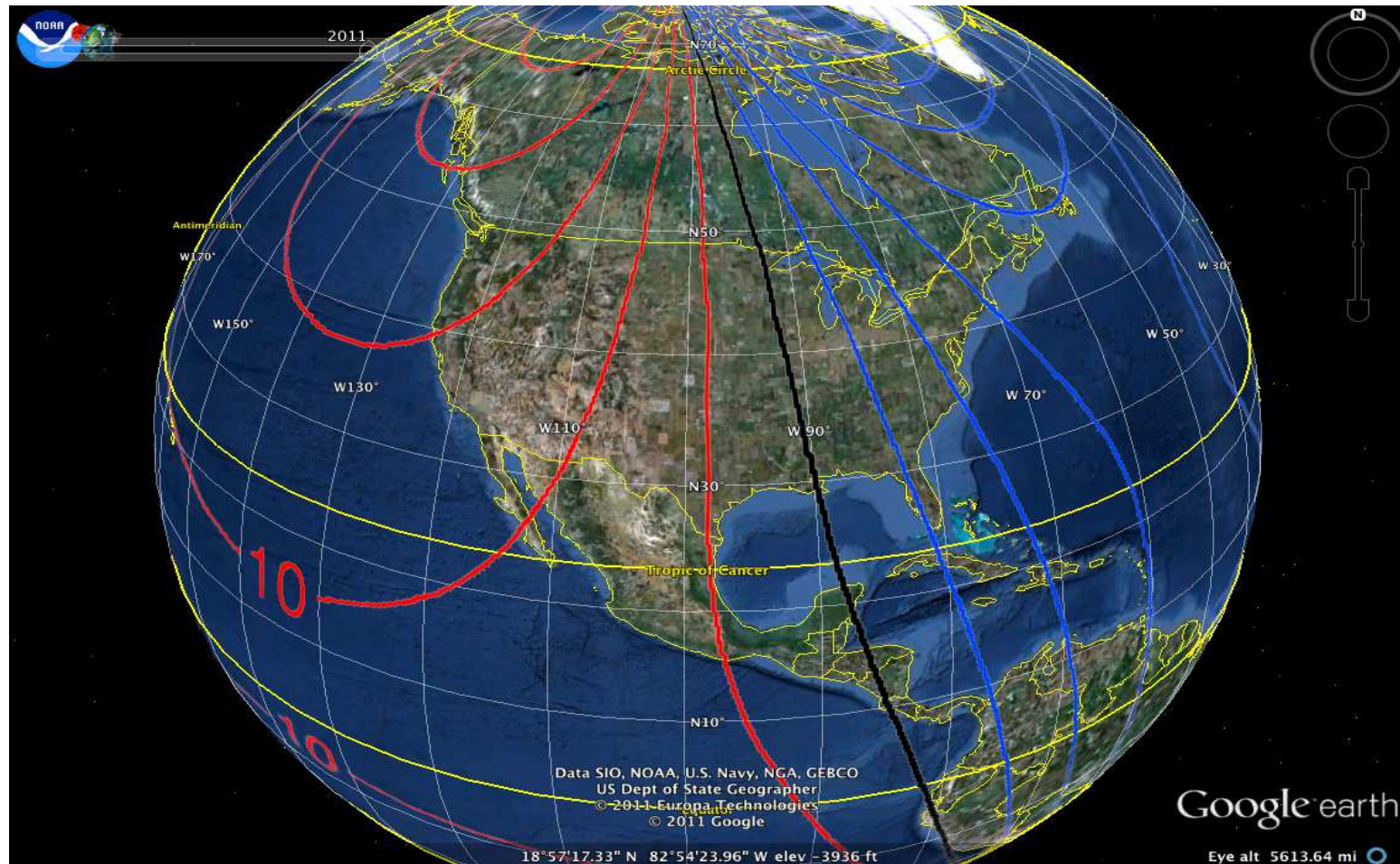


East-west Symmetry (spatial variations from EOF basis function mode 2)

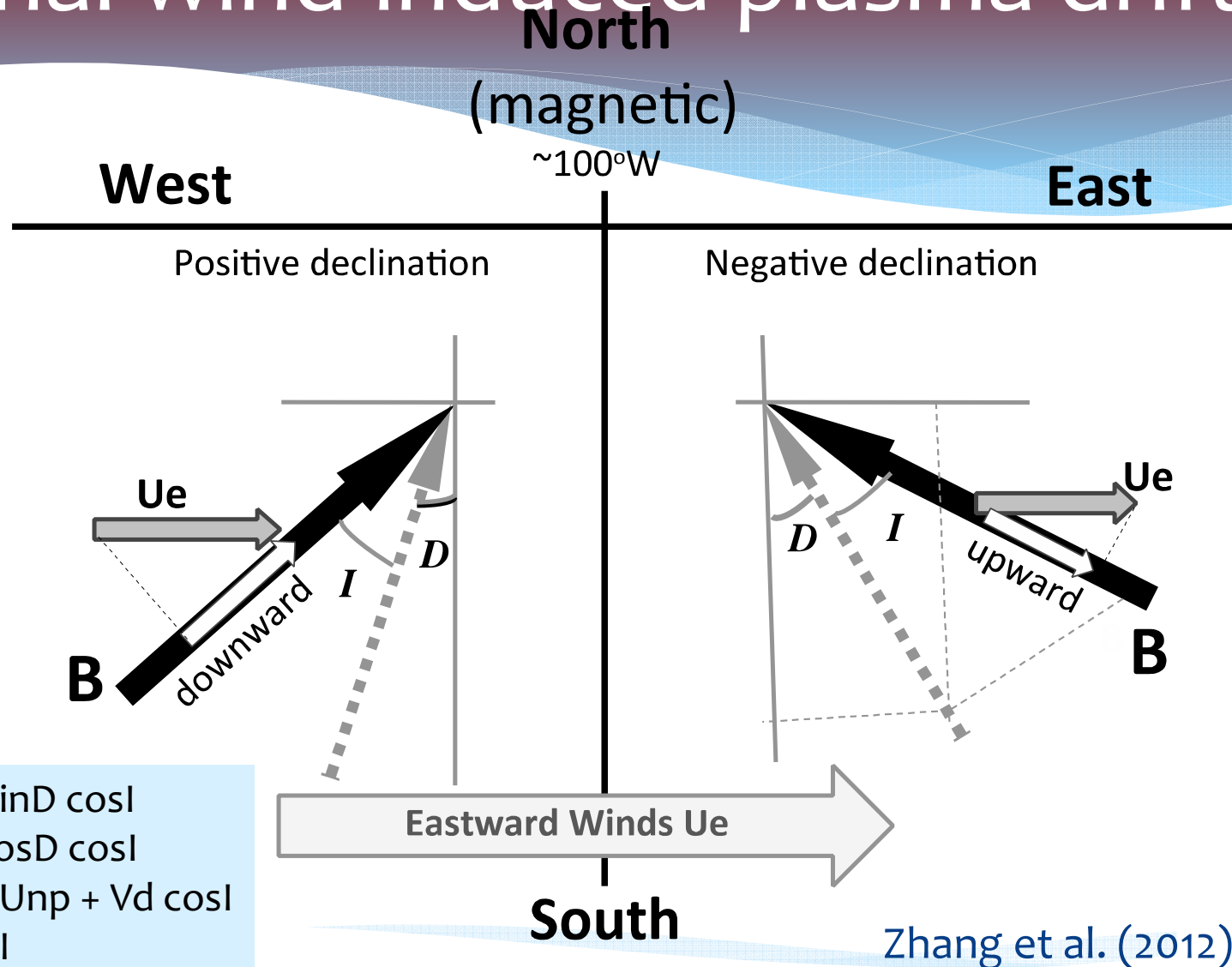


Zhang et al. (JGR, 2013; JGR, 2012; GRL; 2011)

Magnetic declination



Zonal wind-induced plasma drifts



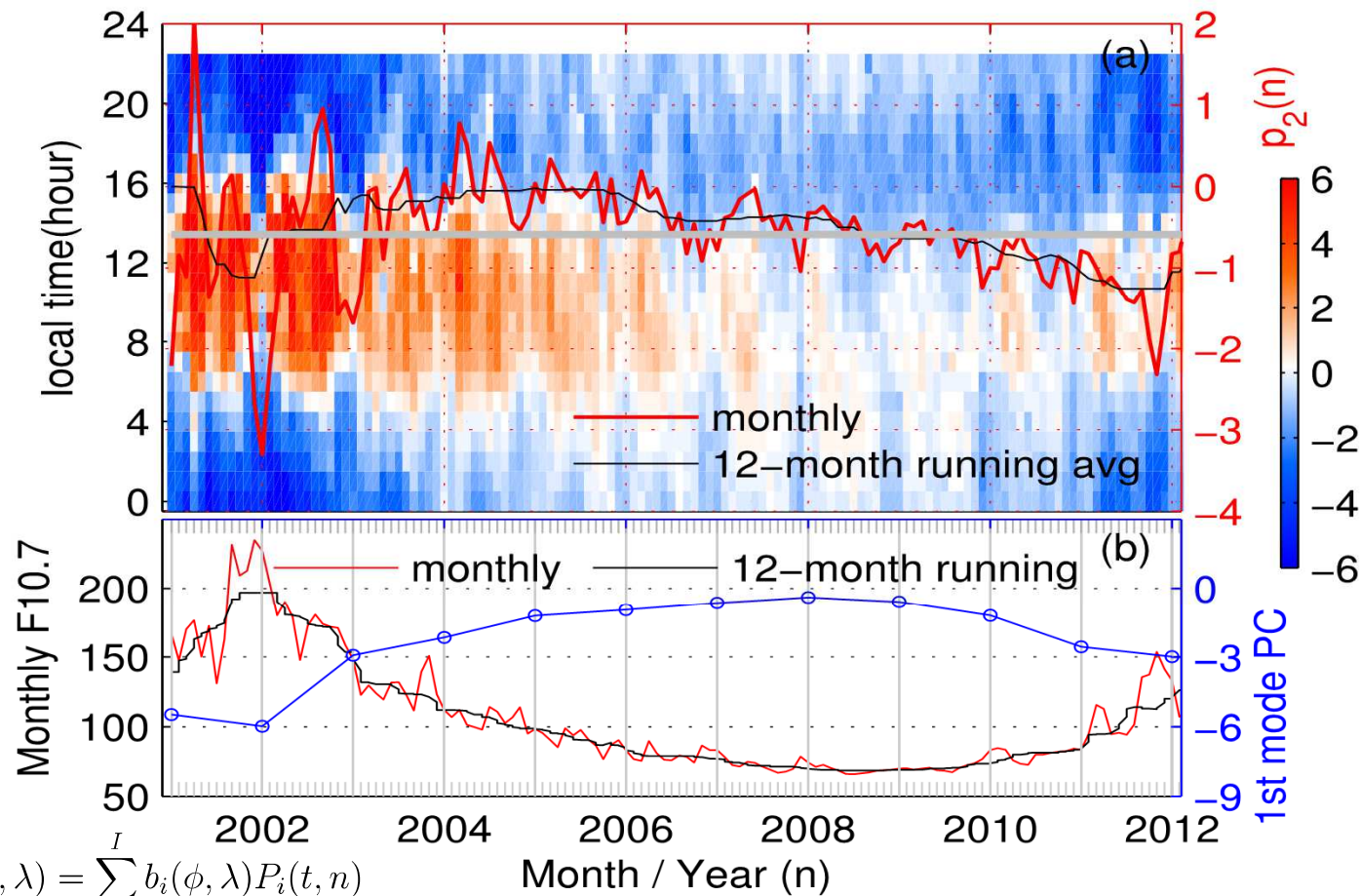
$$U_{ep} = -U_e \sin D \cos I$$

$$U_{np} = U_e \cos D \cos I$$

$$\mathbf{V_p} = U_{ep} + U_{np} + V_d \cos I$$

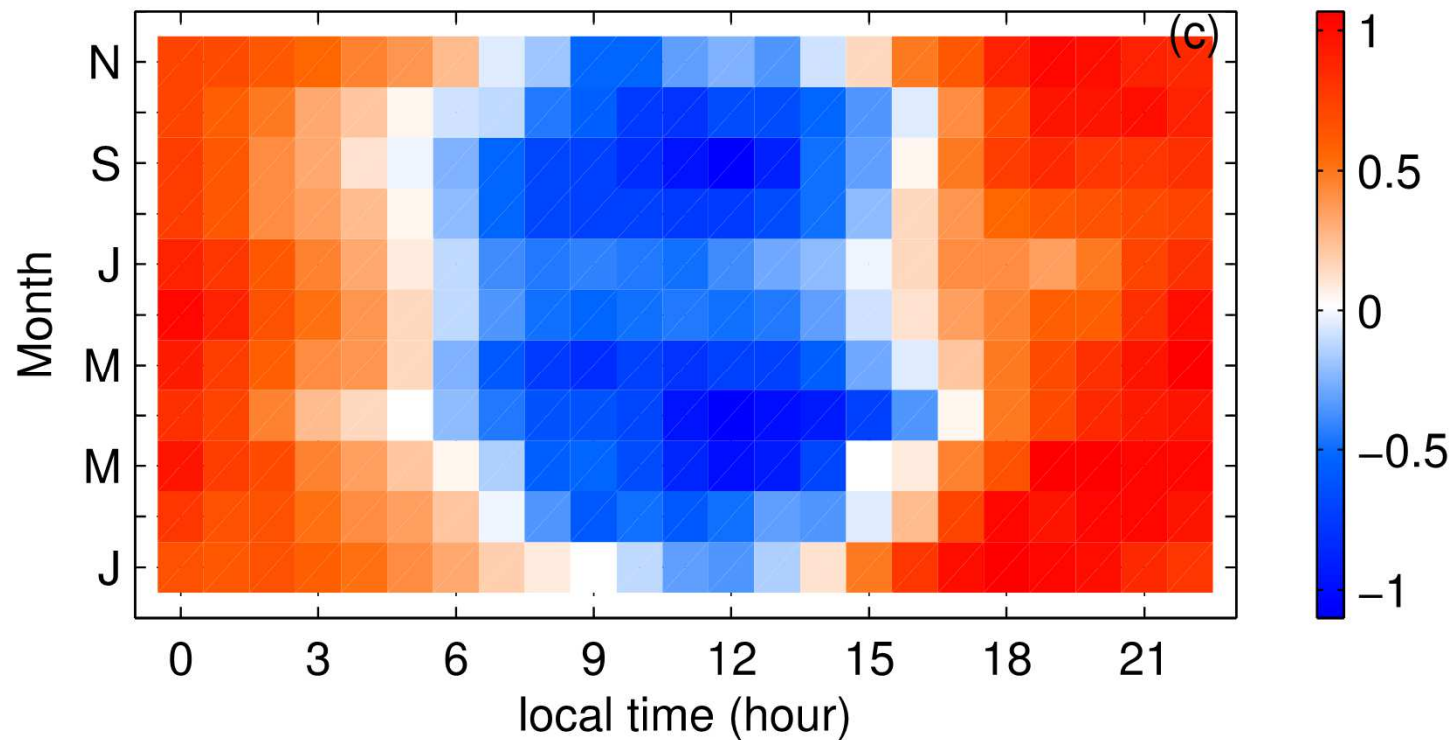
$$V_{zp} = V_p \sin I$$

Temporal variation (EOF model 2)

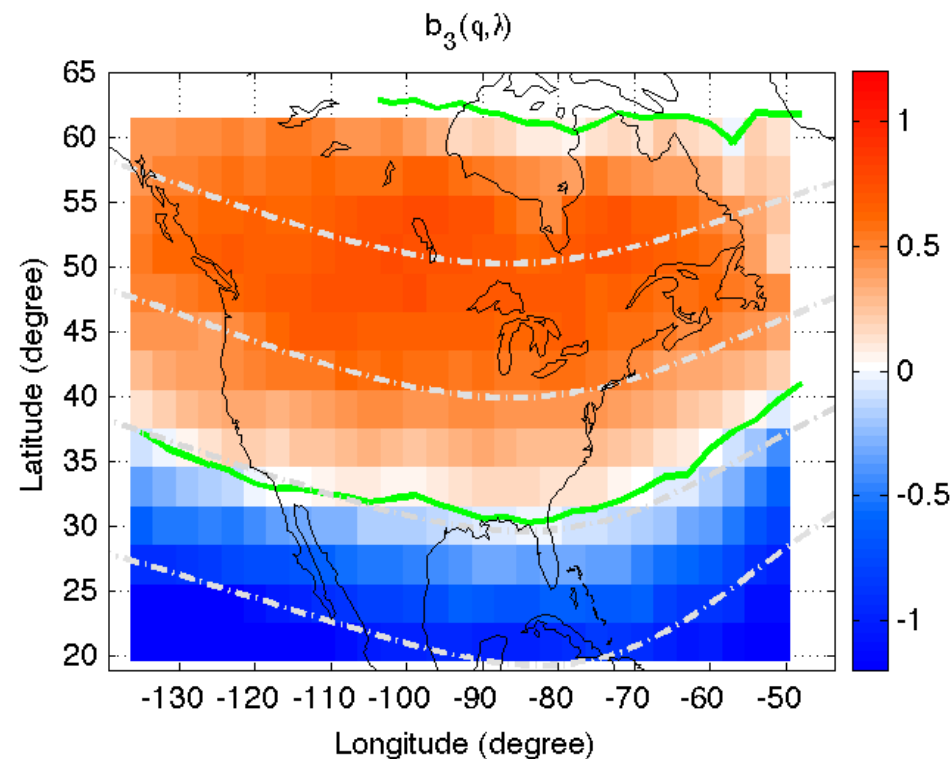


$$T(t, n, \phi, \lambda) = \sum_{i=1}^I b_i(\phi, \lambda) P_i(t, n)$$

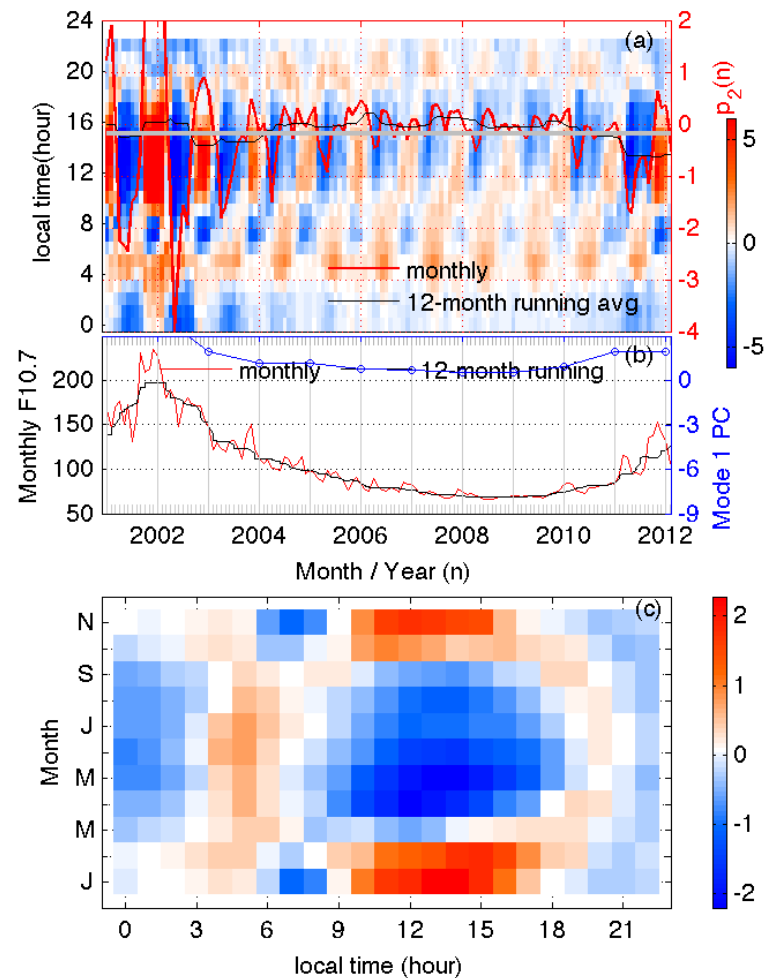
Further decomposed temporal variations (EOF mode 2)



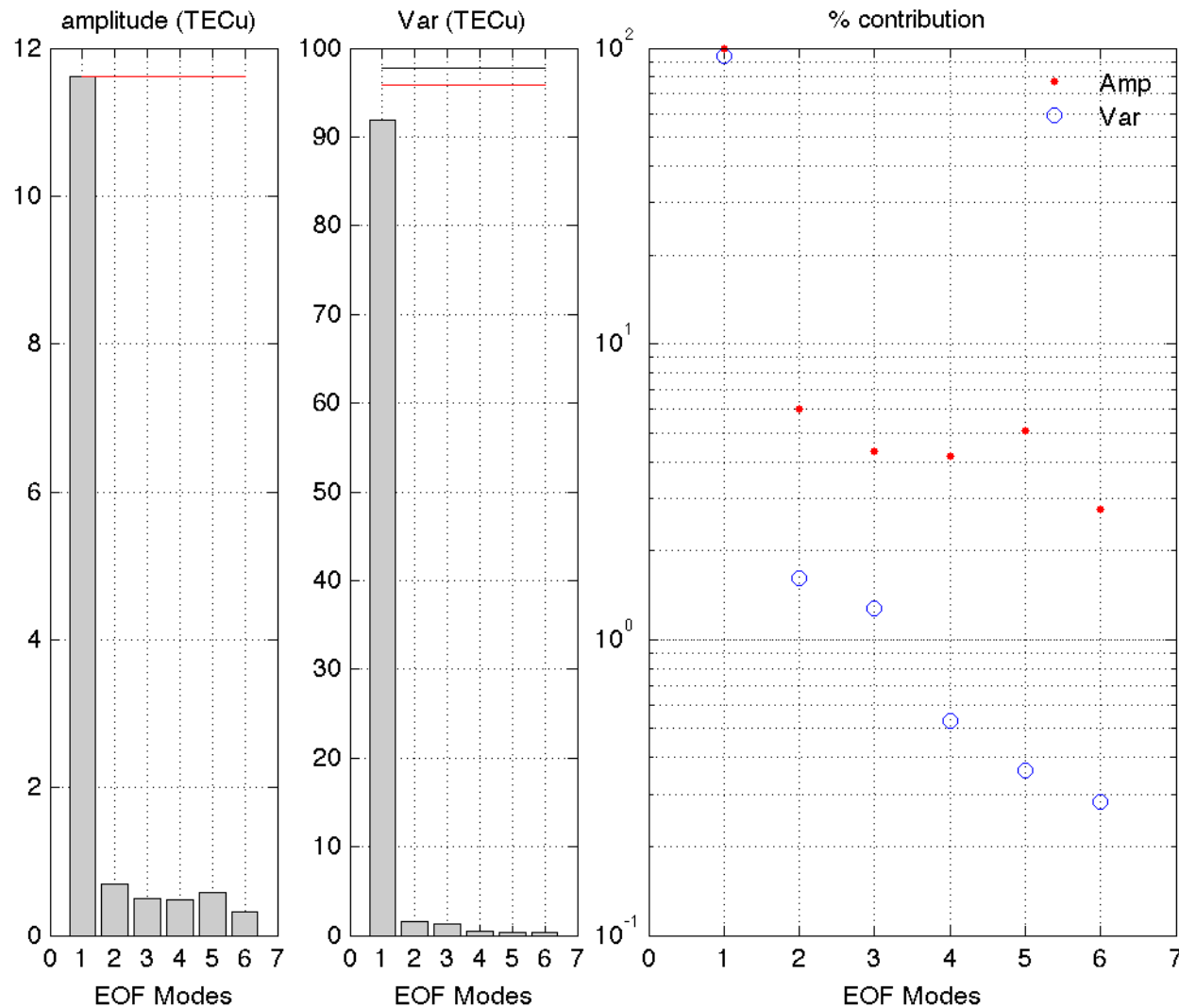
Spatial variations from EOF basis function mode 3



Temporal variations of EOF mode 3

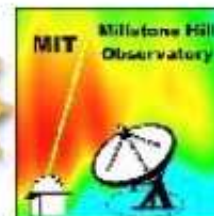


Contributions from various EOF modes





North America Total Electron Content Model



Model

Output Parameter

Day Number (day)	Start 90	End 90	Interval 45	between 1-365
Local Time (hour)	Start 12	End 12	Interval 6	between 0-24
Geodetic Longitude (degree)	Start -13	End -48	Interval 3	between -135 --- -48 E
Geodetic Latitude (degree)	Start 21	End 65	Interval 3	for NATEC: 21-65
Solar Activity	F107 135	Fbar 135	F107: monthly mean; Fbar: 81-day mean between 70-300	
Magnetic Activity	ap index 15		Ap index; between 0-100	

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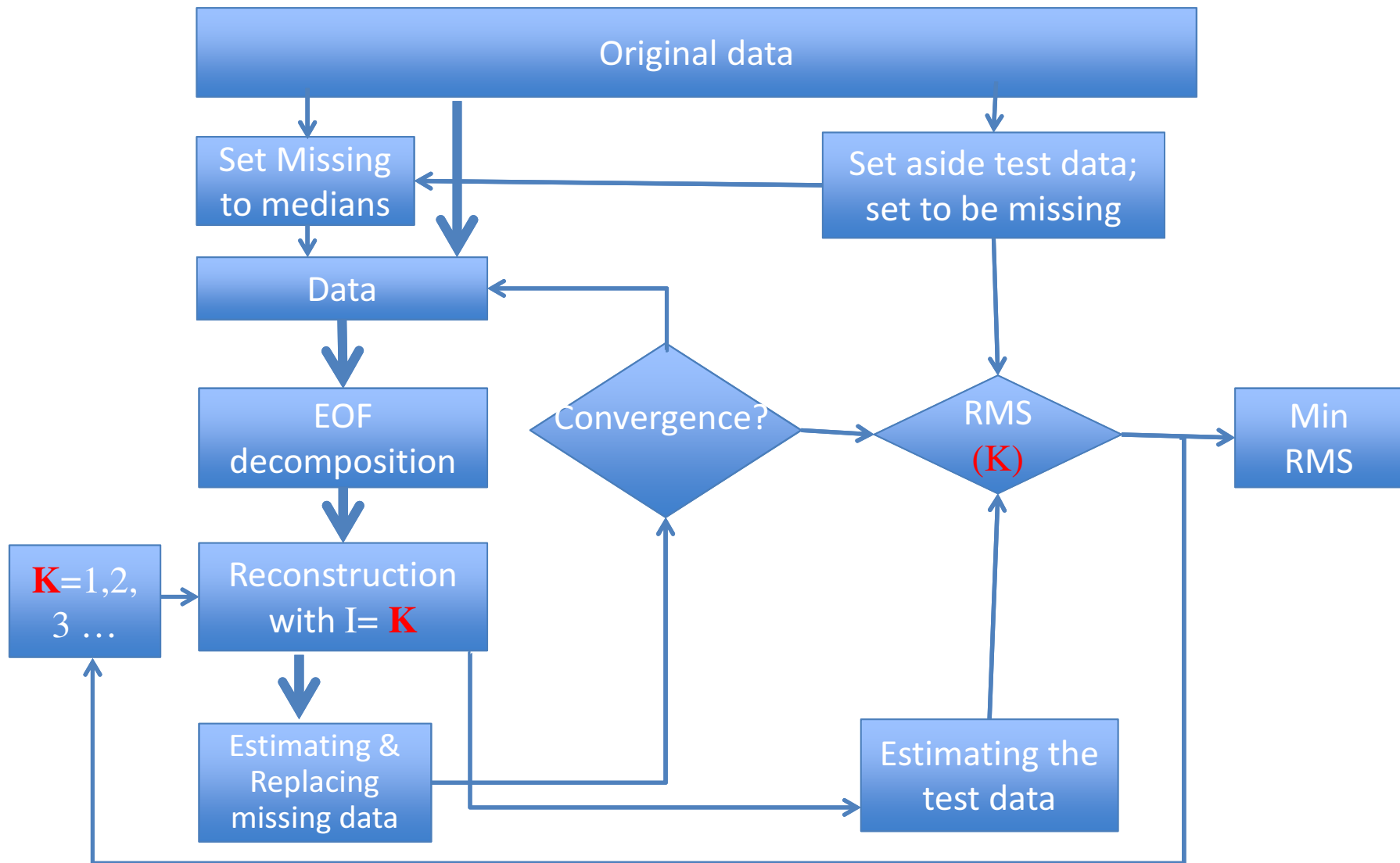
DIEOFs: Data Interpolating Empirical Orthogonal Functions

$$T(t, n, \phi, \lambda) = \sum_{i=1}^I b_i(\phi, \lambda) P_i(t, n) \quad T_{i,j}: i=1,2, \dots, n; j=1:2, \dots, m$$

- The number of modes I equals to n or m ;
- To deal with noisy data and data gaps:
 - When I is truncated to K , the EOF decomposition provides a reconstruction data, smooth and approximated to the original data.
 - If data represented by $T_{i,j}$ is incomplete, an iteration approach may be applied for deriving the best basis functions that may be used to fill in data gaps or even for prediction

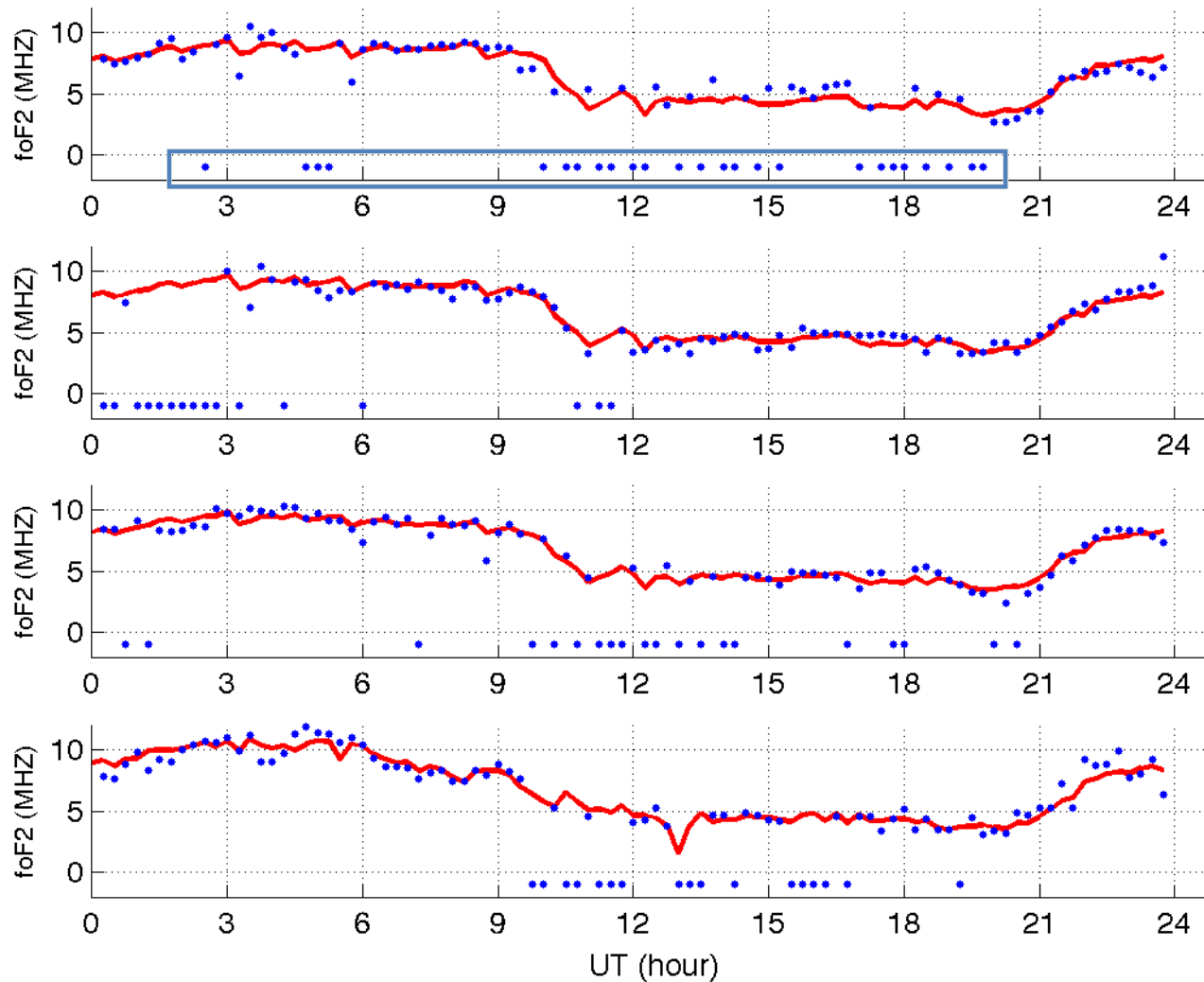
Stoneback et al. (2013)

DIEOF procedure



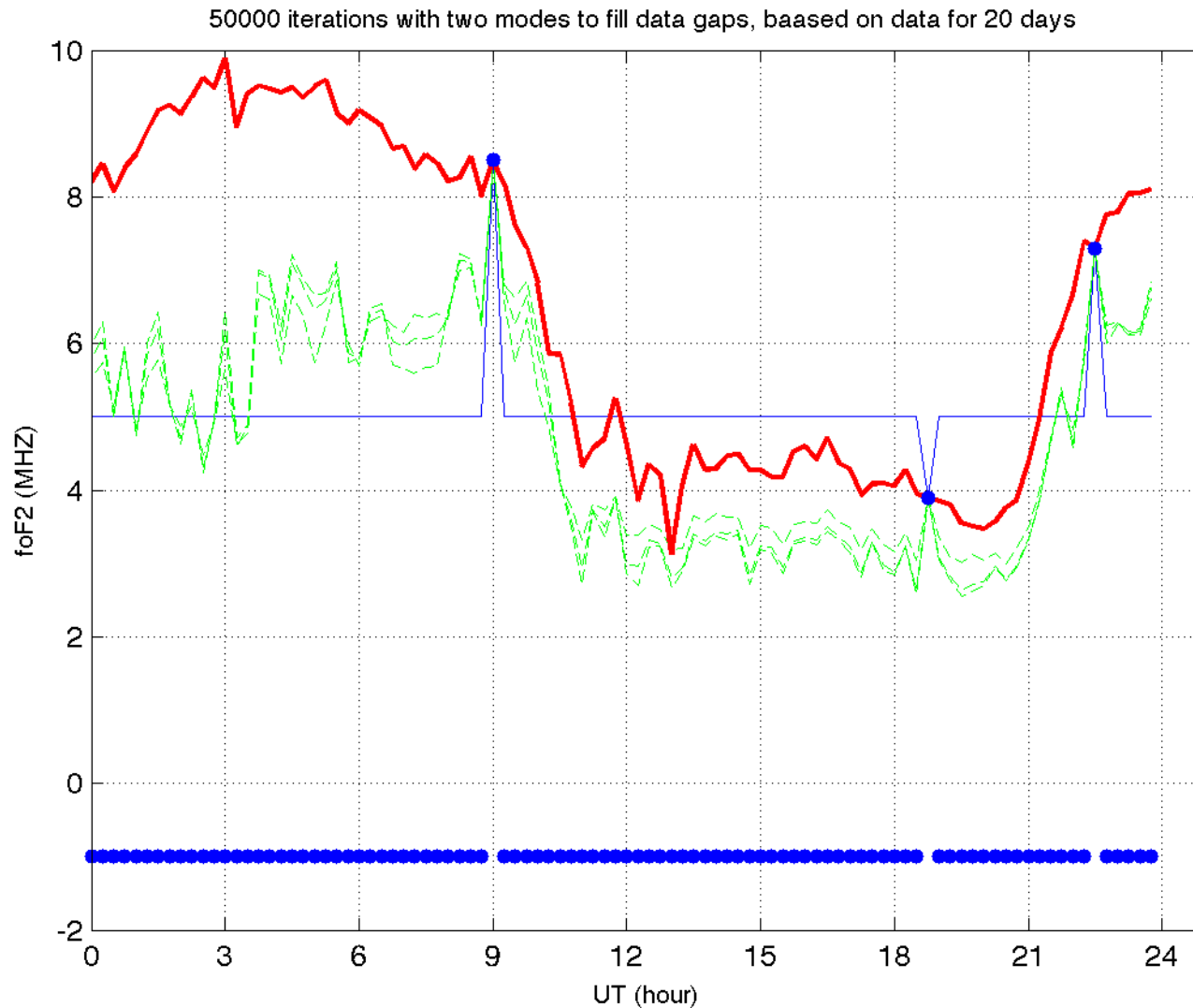
First example

Filling data gaps and smoothing data using EOF modes 1 & 2



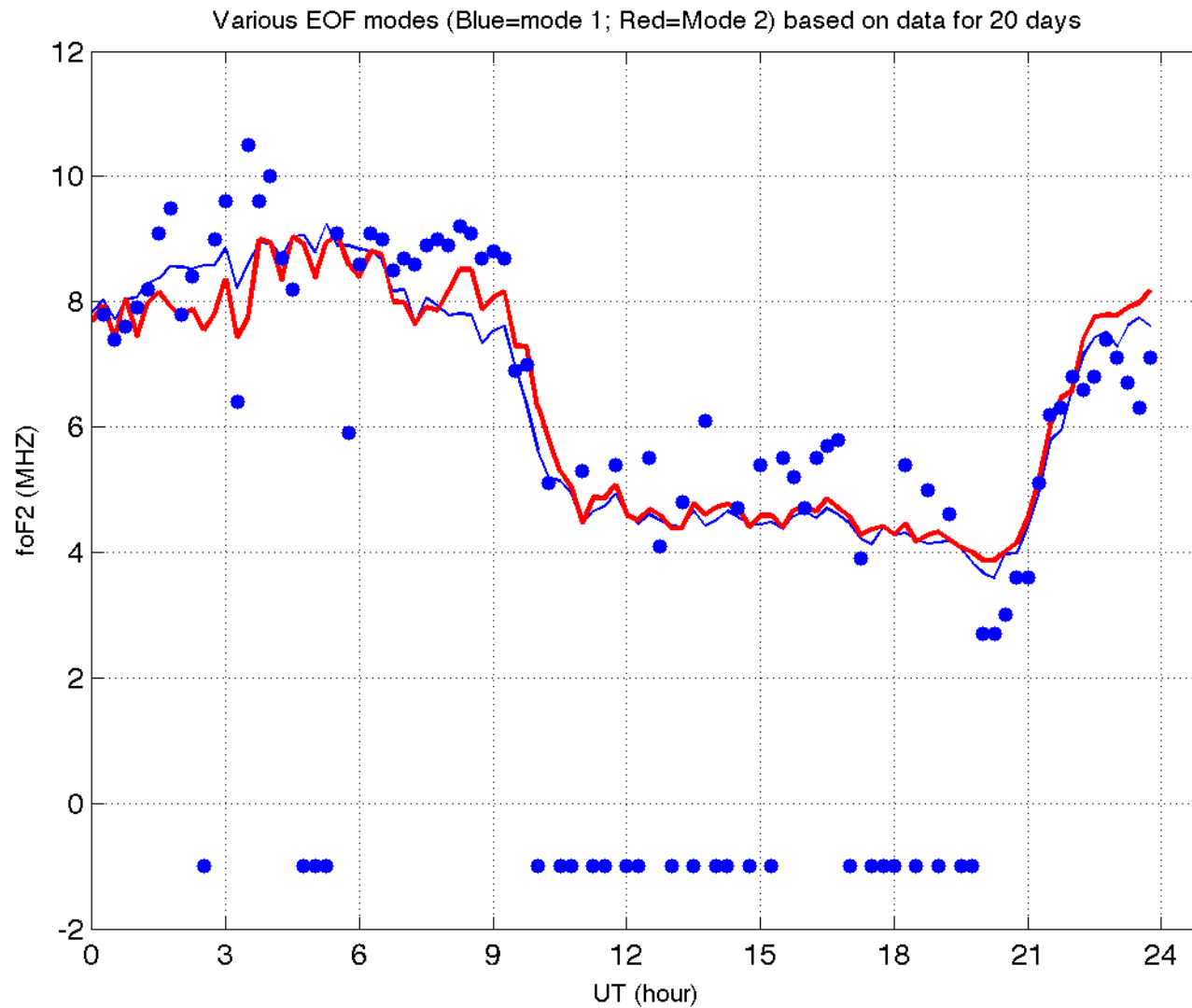
- Single site
- 10 days
- 15 min interval
- Using EOF modes 1 and 2
- After 50,000 iterations
- Shown for days 1, 4, 7, and 10.

Data gaps on day 6

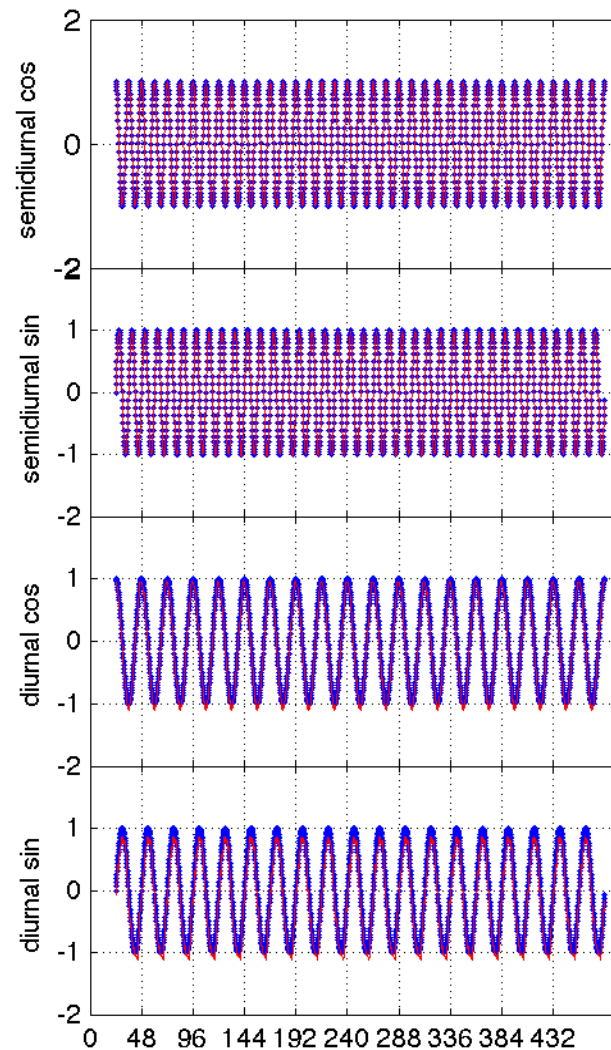
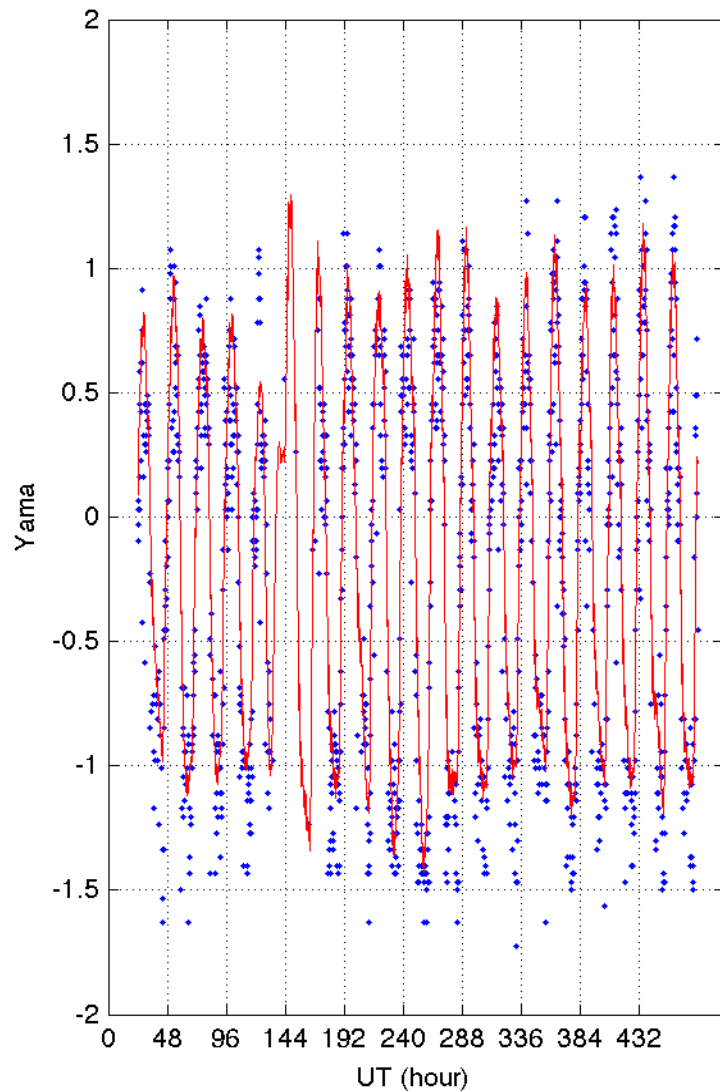


- Only 3 observational points
- Filled the gaps based on DIEOF
- 3 data points were assimilated

Modes 1 and 2

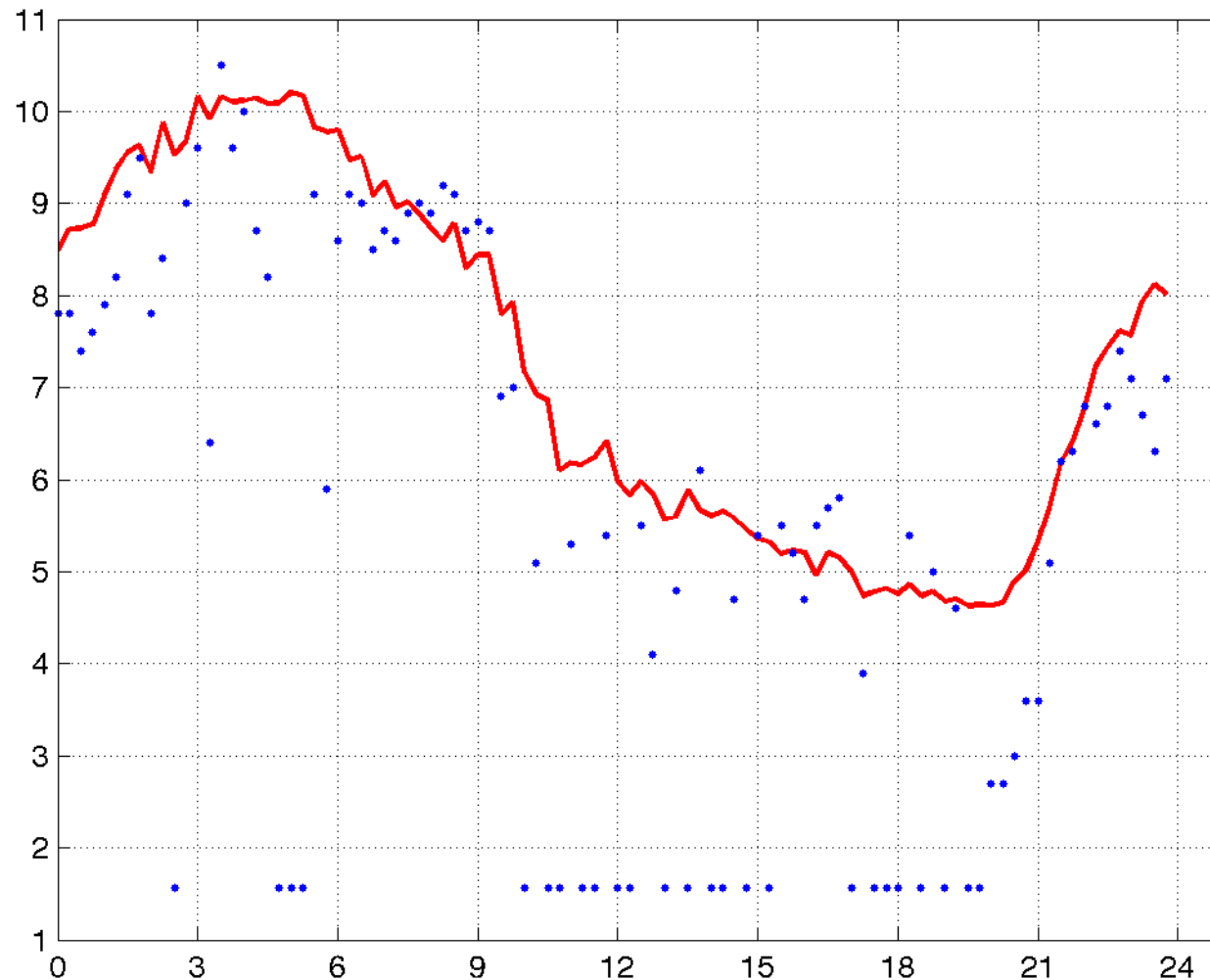


Second example



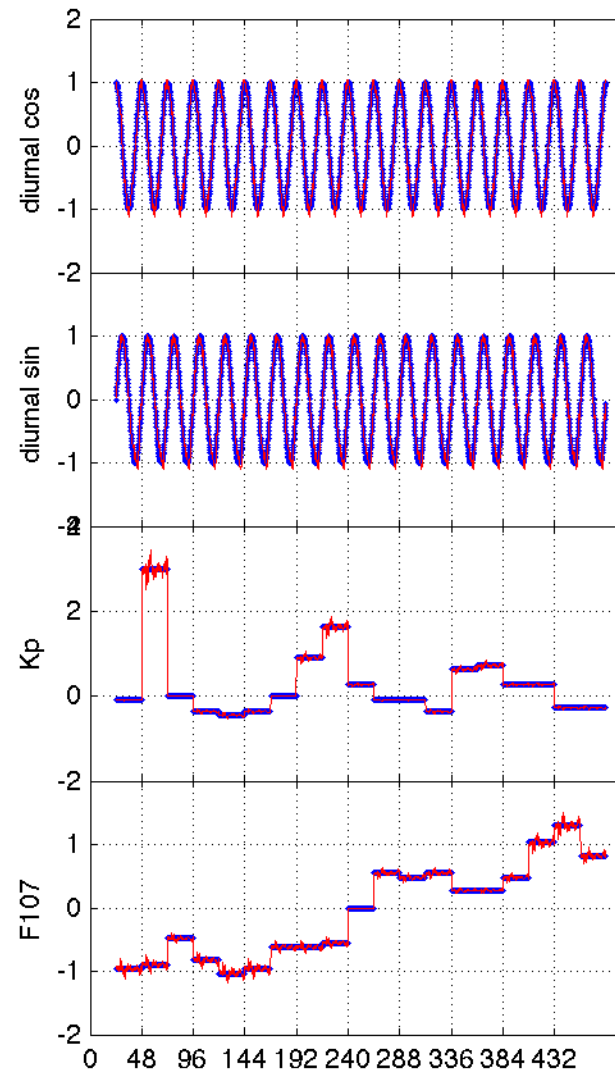
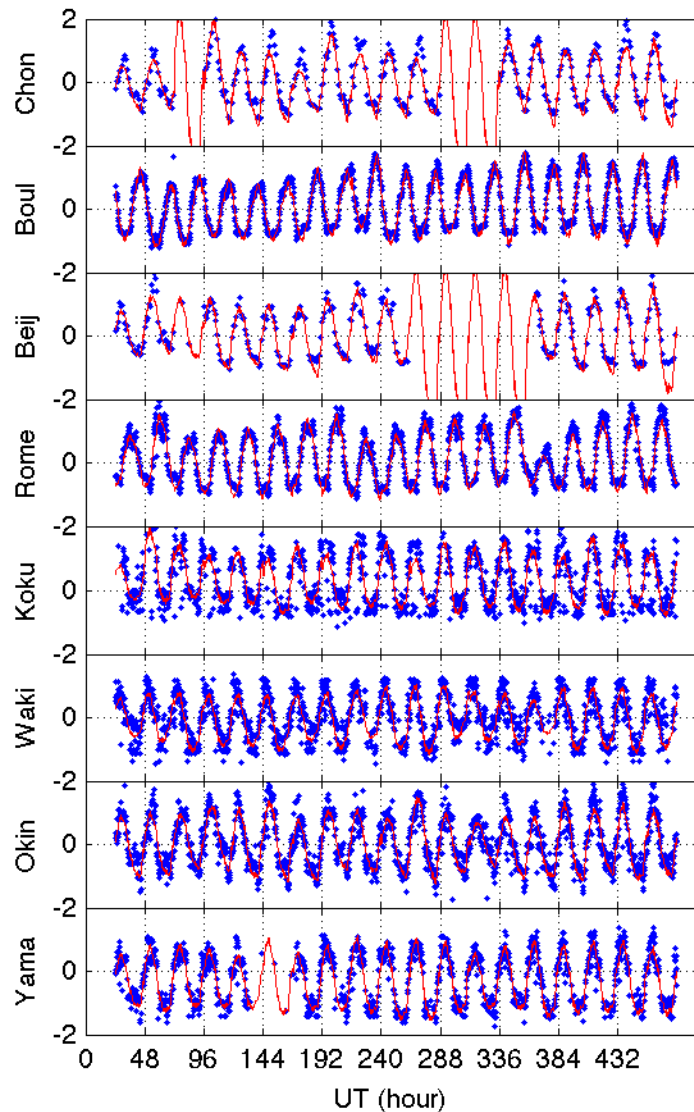
Single site
20 days
Data + 4
physical terms:
(diurnal and
semidiurnal
component)
Fast
convergence
(but
computational
intensive)

Modes are truncated to 4

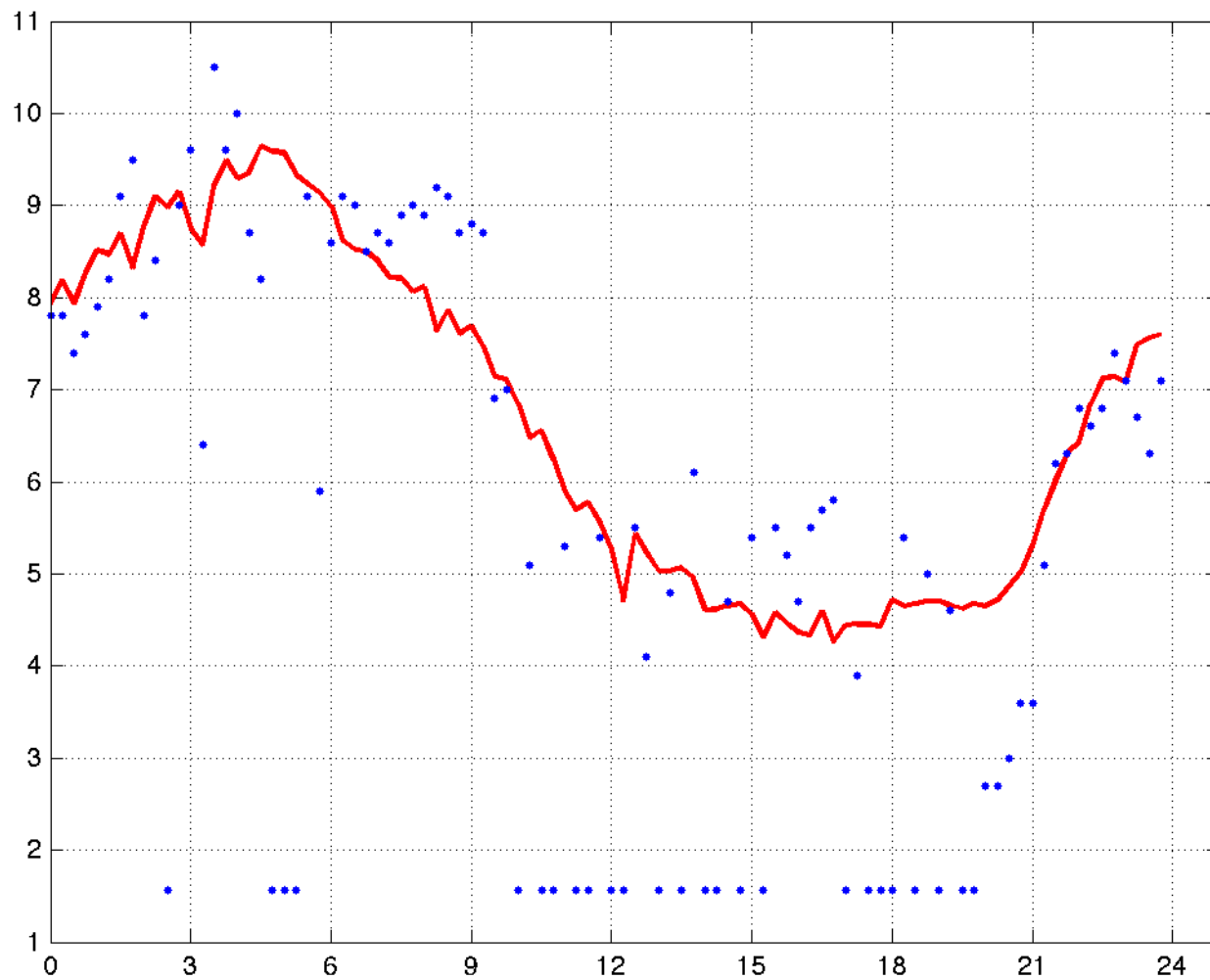


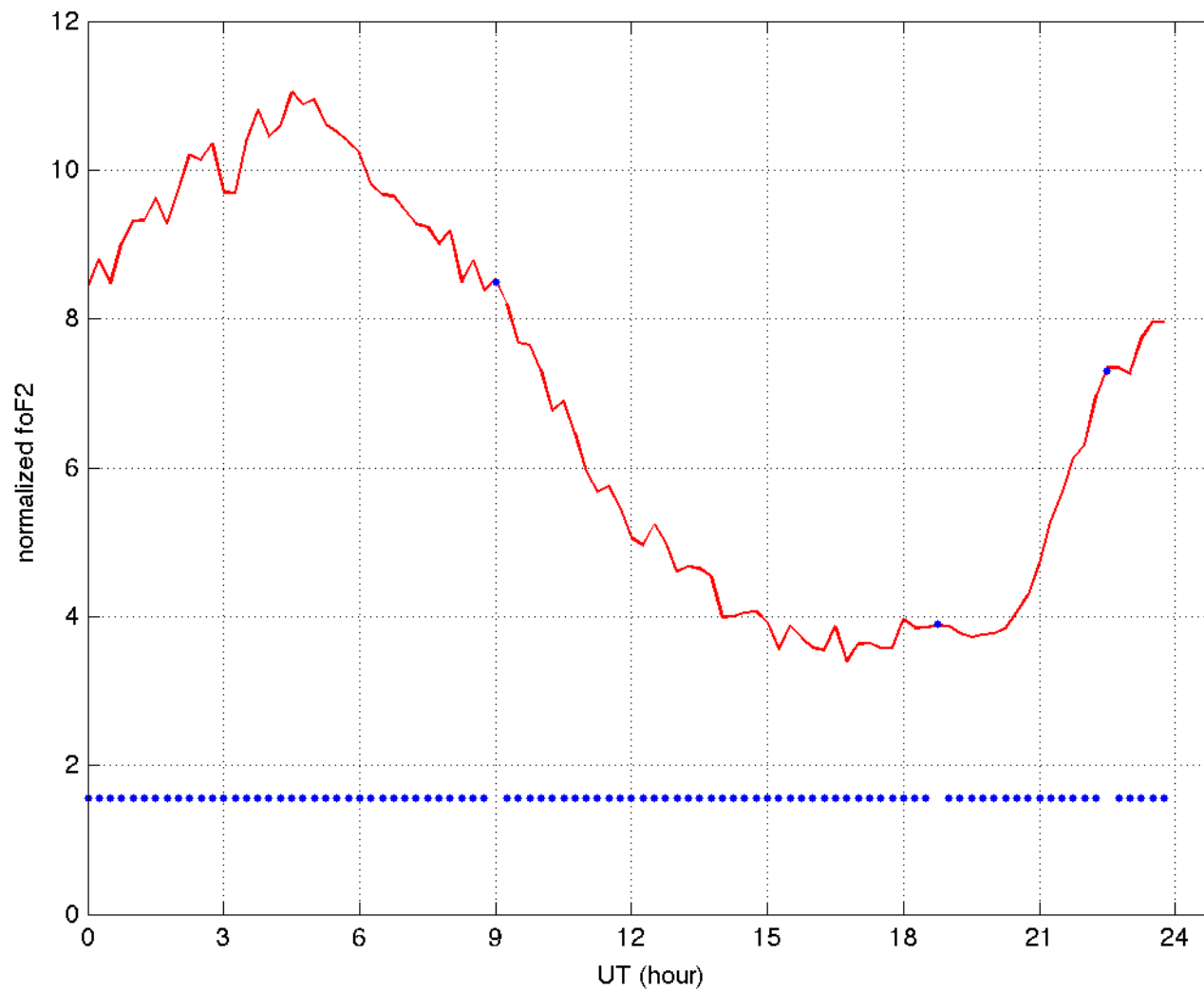
With up to 5000 iterations (if not convergent)

Third example



- 8 sites
- 20 days
- 4 physical terms





Error estimation, etc

- Set aside a portion of the data, and treat these test data as missing
- Go through the entire DIEOF procedure and estimate those test data, and compare these to original data to calculate RMS

Summary

- A powerful tool for modeling and real-time data processing
 - Smoothing data
 - Filling data gaps
- Application for ionospheric predication
 - Using statistical correlation between each of EOF basis functions with geophysical parameters
 - Using EOF basis functions to fill “future data gaps” for predication

Real-time IRI @ MIT

