

Assimilative Real-Time Mapping of the F2 Peak Using Elastic Linear Optimization (ELO) Method

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Outline

- **Review of the CCIR/URSI mapping technique of the F2 layer peak.** The medians of foF2 is a function of latitude (λ), longitude (φ), and universal time (t), expanded in a series with 988 coefficients.
 - How is the expansion done?
 - What are the meaning of the coefficients?
- **Mapping in real time and the Elastic Linear Optimization (ELO) Method.**
 - Applied to the measured data for any given time.
 - ELO method – to adjust the coefficients to do real time mapping.
- **Examples.**
- **Discussion.**

Review of the CCIR/URSI Mapping Technique

- The IRI electron density profile is one of the best available global ionospheric density models, but it is a monthly median model.
- The peak density and height of the $F2$ layer are the most important parameters.
- Review of the current CCIR mapping technique: The median of $foF2$ is a function of universal time (t), latitude (λ), longitude (φ), and sunspot number (R), and it is expanded in the form of series based on physical understanding and synoptic studies [Jones and Gallet, 1965].

$$foF2 = f0F2(t, \lambda, \varphi, R)$$

Review of the CCIR/URSI Mapping Technique -- Diurnal Variation

- Time dependence:

Solar control \Rightarrow Diurnal variation

$$foF2 = a_0(\lambda, \varphi, R) + \sum_{i=1}^I \left[a_{2i-1}(\lambda, \varphi, R) \sin(iT) + a_{2i}(\lambda, \varphi, R) \cos(iT) \right] \quad (1)$$

t = universal time, hour of day, $0 \leq t \leq 24$

$$T = 15t - 180 \text{ (degrees)} : \begin{cases} T = 0, & \text{at noon time, } t = 12 \\ T = 180, & \text{at midnight, } t = 24 \end{cases}$$

Review of the CCIR/URSI Mapping Technique

-- Locational Variation

- Geographic location dependence:
 - Solar control \Rightarrow Location variations

$$a_i(\lambda, \varphi, R) = b_{i,0} + \sum_{j=1}^J \left[b_{i,j} \cos(j\varphi) + c_{i,j} \sin(j\varphi) \right] \cos^j \lambda, \quad i = 0, 1, 2, \dots, 2I \quad (2a)$$

$$\begin{cases} a_0(\lambda, \varphi, R) = b_{0,0} + \sum_{j=1}^J \left[b_{0,j} \cos(j\varphi) + c_{0,j} \sin(j\varphi) \right] \cos^j \lambda \\ a_{2i-1}(\lambda, \varphi, R) = b_{2i-1,0} + \sum_{j=1}^J \left[b_{2i-1,j} \cos(j\varphi) + c_{2i-1,j} \sin(j\varphi) \right] \cos^j \lambda \\ a_{2i}(\lambda, \varphi, R) = b_{2i,0} + \sum_{j=1}^J \left[b_{2i,j} \cos(j\varphi) + c_{2i,j} \sin(j\varphi) \right] \cos^j \lambda \end{cases} \quad (2b)$$

Review of the CCIR/URSI Mapping Technique

- Replacing $a_i(\lambda, \varphi)$ by equation (2b),

$$foF2 = a_0(\lambda, \varphi, R) + \sum_{i=1}^I [a_{2i-1}(\lambda, \varphi, R) \sin(iT) + a_{2i}(\lambda, \varphi, R) \cos(iT)] \quad (1)$$

$$a_i(\lambda, \varphi, R) = b_{i,0} + \sum_{j=1}^J [b_{i,j} \cos(j\varphi) + c_{i,j} \sin(j\varphi)] \cos^j \lambda, \quad i = 0, 1, 2, \dots, 2I \quad (2a)$$

$$\begin{aligned} foF2 = & b_{0,0} + \sum_{j=1}^J [b_{0,j} \cos(j\varphi) \cos^j \lambda + c_{0,j} \sin(j\varphi) \cos^j \lambda] \\ & + \sum_{i=1}^I \left[\left\{ b_{2i-1,0} + \sum_{j=1}^J [b_{2i-1,j} \cos(j\varphi) \cos^j \lambda + c_{2i-1,j} \sin(j\varphi) \cos^j \lambda] \right\} \sin(iT) \right. \\ & \left. + \left\{ b_{2i,0} + \sum_{j=1}^J [b_{2i,j} \cos(j\varphi) \cos^j \lambda + c_{2i,j} \sin(j\varphi) \cos^j \lambda] \right\} \cos(iT) \right] \end{aligned} \quad (3)$$

Review of the CCIR/URSI Mapping Technique

-- Geomagnetic Correction

- Location dependence:
 - Symmetrical about the geographic equator? No.
 - Why not using the geomagnetic coordinates?
 - If using geomagnetic coordinates instead of geographic coordinates, more terms are required to describe sun rising and setting features.
 - The geomagnetic coordinates change from one year to another.
- It needs geomagnetic location correction.

$$\begin{aligned}
 b_{0,0} &= \sum_{k=0}^{K(0)} U_{0,0,k} \sin^k \chi, \quad b_{2i-1,0} = \sum_{k=0}^{K(0)} U_{2i-1,0,k} \sin^k \chi, \quad b_{2i,0} = \sum_{k=0}^{K(0)} U_{2i,0,k} \sin^k \chi, \quad (i=1,2,\dots,I) \\
 b_{0,j} &= \sum_{k=0}^{K(j)} U_{0,j,2k} \sin^k \chi, \quad c_{0,j} = \sum_{k=0}^{K(j)} U_{0,j,2k+1} \sin^k \chi, \quad (j=1,2,\dots,J) \\
 \left\{ \begin{aligned}
 b_{2i-1,j} &= \sum_{k=0}^{K(j)} U_{2i-1,j,2k} \sin^k \chi, \quad b_{2i,j} = \sum_{k=0}^{K(j)} U_{2i,j,2k} \sin^k \chi \\
 c_{2i-1,j} &= \sum_{k=0}^{K(j)} U_{2i-1,j,2k+1} \sin^k \chi, \quad c_{2i,j} = \sum_{k=0}^{K(j)} U_{2i,j,2k+1} \sin^k \chi
 \end{aligned} \right. \quad \begin{pmatrix} i=1,2,\dots,I \\ j=1,2,\dots,J \end{pmatrix} \quad (4)
 \end{aligned}$$

Review of the CCIR/URSI Mapping Technique

$$\begin{aligned}
 foF2 = & \sum_{k=0}^{K(0)} U_{0,0,k} \sin^k \chi + \sum_{j=1}^J \sum_{k=0}^{K(j)} \left[U_{0,j,2k} \cos(j\varphi) \cos^j \lambda + U_{0,j,2k+1} \sin(j\varphi) \cos^j \lambda \right] \sin^k \chi \\
 & + \sum_{i=1}^I \sum_{k=0}^{K(0)} \left[U_{2i-1,0,k} \sin(iT) + U_{2i,0,k} \cos(iT) \right] \sin^k \chi \\
 & + \sum_{i=1}^I \sum_{j=1}^J \sum_{k=0}^{K(j)} \left[U_{2i-1,j,2k} \sin(iT) \cos(j\varphi) \cos^j \lambda + U_{2i,j,2k} \cos(iT) \cos(j\varphi) \cos^j \lambda \right. \\
 & \quad \left. + U_{2i-1,j,2k+1} \sin(iT) \sin(j\varphi) \cos^j \lambda + U_{2i,j,2k+1} \cos(iT) \sin(j\varphi) \cos^j \lambda \right] \sin^k \chi
 \end{aligned}$$

Note: (1) A set of The coefficients for a month is a 3-dimensional uneven array.

$$U(i, j, k), \quad i \in [0, 2I], \quad j \in [0, J], \quad k \in [0, 2K + 1]$$

$$I = 6, \quad J = 8,$$

$K(j)$ is a set of integers depending on j :

$$\begin{cases} j = & 0, & 1, & 2, & 3, & 4, & 5, & 6, & 7, & 8, & \dots \\ K(j) = & 11, & 11, & 8, & 4, & 1, & 0, & 0, & 0, & 0, & \dots \end{cases}$$

(2) $U(i, j, k)$: index i to give the diurnal variation

index j to give the geographic location variation

index k to give the geomagnetic correction

(3) The coefficients are given for 12 months and normalized with the sunspot number $R = 0$ and $R = 100$.

$$(4) \sum N_j = 12 + 24 + 18 + 10 + 4 + 2 + 2 + 2 + 2 = 76$$

$$\text{Total number of coefficients} = 13 \times 76 = 988.$$

Basic Formula

$$foF2(t, \varphi, \lambda, \chi) = \sum_{k=0}^{K(0)} G_{0k}(\lambda, \chi) \left[U_{0,0,k} + \sum_{i=1}^I \left(\begin{matrix} U_{2i-1,0,k} \sin(iT) \\ + U_{2i,0,k} \cos(iT) \end{matrix} \right) \right] + \sum_{j=1}^J \sum_{k=0}^{K(j)} \left[\begin{matrix} G_{jk}(\lambda, \chi) \cos(j\varphi) \left[U_{0,j,2k} + \sum_{i=1}^I \left(\begin{matrix} U_{2i-1,j,2k} \sin(iT) \\ + U_{2i,j,2k} \cos(iT) \end{matrix} \right) \right] \\ + G_{jk}(\lambda, \chi) \sin(j\varphi) \left[U_{0,j,2k+1} + \sum_{i=1}^I \left(\begin{matrix} U_{2i-1,j,2k+1} \sin(iT) \\ + U_{2i,j,2k+1} \cos(iT) \end{matrix} \right) \right] \end{matrix} \right]$$

where $G_{jk}(\lambda, \chi) = \cos^j \lambda \sin^k \chi$ and $\{U_{i,j,k}\}$ is a set of CCIR/URSI coefficients for a month.

This is the formulae system used in the current IRI program

$$foF2 = \sum_{k=0}^{K(0)} G_{0k}(\lambda, \chi) D_{0,k} + \sum_{j=1}^J \sum_{k=0}^{K(j)} \left[G_{jk}(\lambda, \chi) (D_{j,2k} \cos(j\varphi) + D_{j,2k+1} \sin(j\varphi)) \right]$$

$$G_{jk}(\lambda, \chi) = \cos^j \lambda \sin^k \chi$$

$$D_{0,k} = U_{0,0,k} + \sum_{i=1}^I \left(\begin{matrix} U_{2i-1,0,k} \sin(iT) \\ + U_{2i,0,k} \cos(iT) \end{matrix} \right), \quad j = 0; \quad k = 0, 1, 2, \dots, K(j)$$

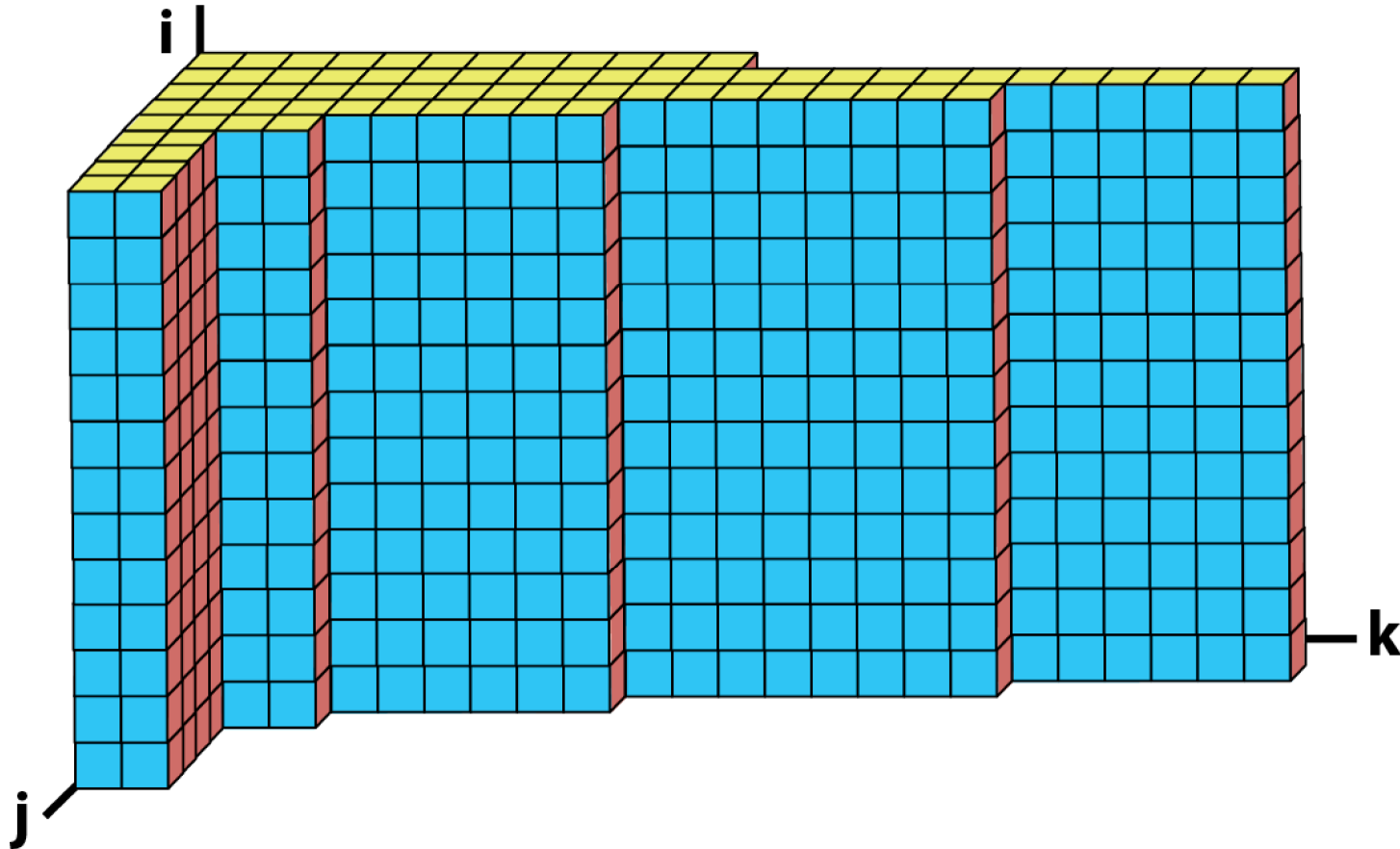
$$D_{j,2k} = U_{0,j,2k} + \sum_{i=1}^I \left(\begin{matrix} U_{2i-1,j,2k} \sin(iT) \\ + U_{2i,j,2k} \cos(iT) \end{matrix} \right), \quad j = 1, 2, \dots, J; \quad k = 0, 1, 2, \dots, K(j)$$

$$D_{j,2k+1} = U_{0,j,2k+1} + \sum_{i=1}^I \left(\begin{matrix} U_{2i-1,j,2k+1} \sin(iT) \\ + U_{2i,j,2k+1} \cos(iT) \end{matrix} \right), \quad j = 1, 2, \dots, J; \quad k = 0, 1, 2, \dots, K(j)$$

Data structure of coefficients

$$foF2 = \sum_{k=0}^{K(0)} \sin^k \chi \left[U_{0,0,k} + \sum_{i=1}^I \left(U_{2i-1,0,k} \sin(iT) \right) + \sum_{j=1}^J \sum_{k=0}^{K(j)} \left[\cos(j\varphi) \cos^j \lambda \sin^k \chi \left[U_{0,j,2k} + \sum_{i=1}^I \left(U_{2i-1,j,2k} \sin(iT) \right) + \sin(j\varphi) \cos^j \lambda \sin^k \chi \left[U_{0,j,2k+1} + \sum_{i=1}^I \left(U_{2i-1,j,2k+1} \sin(iT) \right) \right] \right] \right] \right]$$

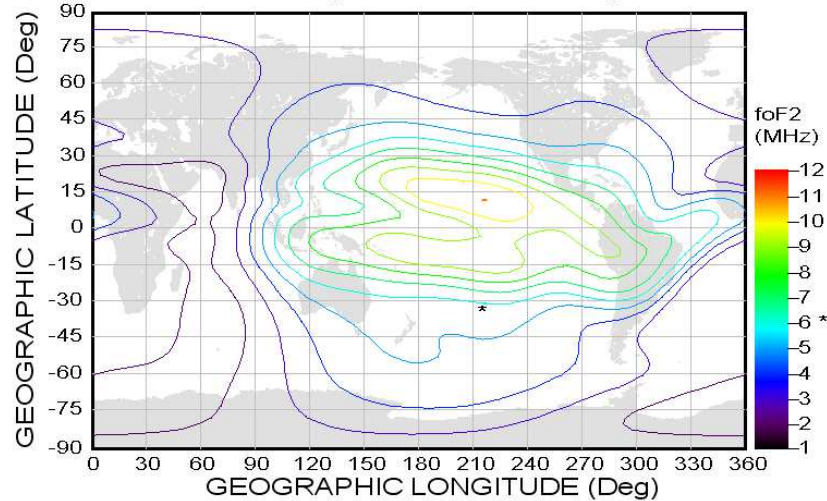
$$i = (0, I), I = 6; \quad j = (0, J), J = 8; \quad k = (0, K(j)), \begin{cases} j & = & 0, & 1, & 2, & 3, & 4, & 5, & 6, & 7, & 8 \\ K(j) & = & 11, & 11, & 8, & 4, & 1, & 0, & 0, & 0, & 0 \end{cases}$$



Examples of Numerical Mapping foF2

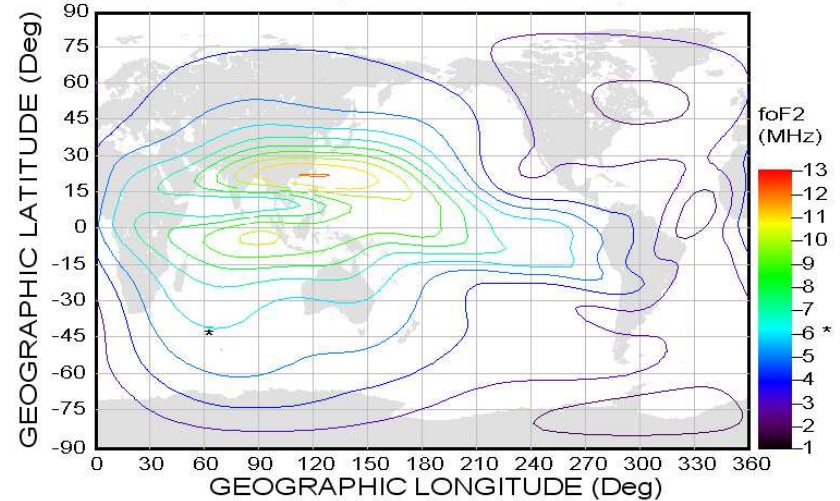
foF2 (MHz) ~ LONGITUDE (Deg) and LATITUDE (Deg)

UT 00:00 Year:2008 Month: 4 Day:15 (106/Year)
SSN: 3.3 IG: -7.5 (OBSERVED for the month)



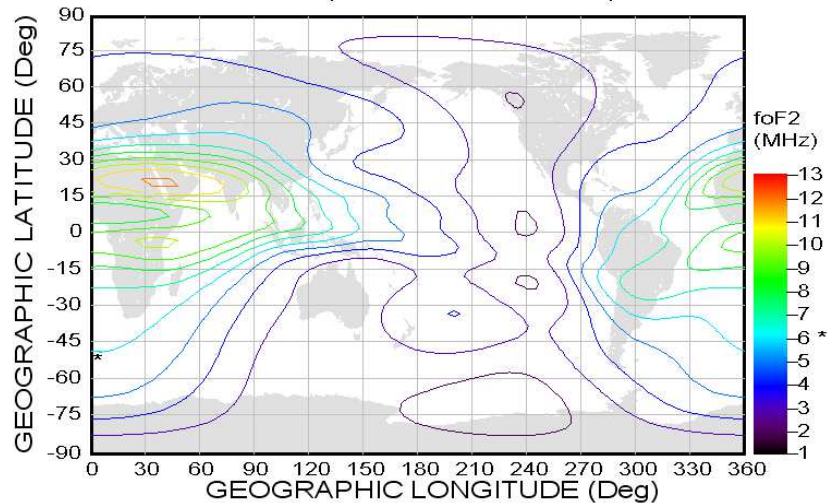
foF2 (MHz) ~ LONGITUDE (Deg) and LATITUDE (Deg)

UT 06:00 Year:2008 Month: 4 Day:15 (106/Year)
SSN: 3.3 IG: -7.5 (OBSERVED for the month)



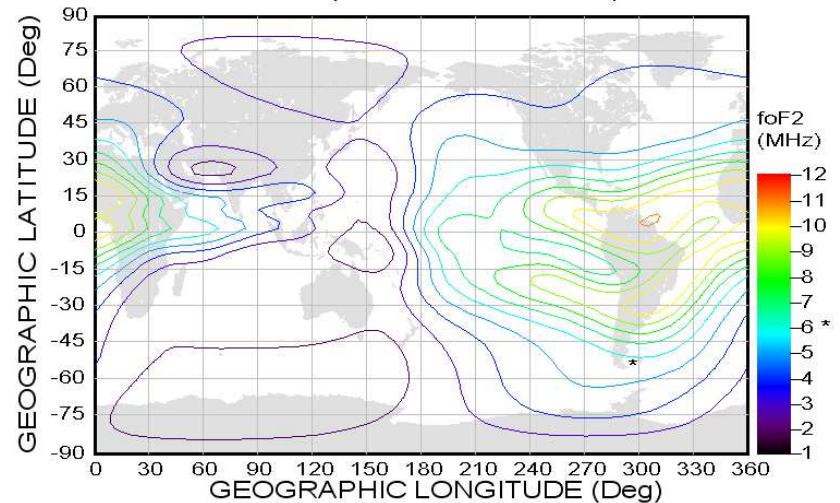
foF2 (MHz) ~ LONGITUDE (Deg) and LATITUDE (Deg)

UT 12:00 Year:2008 Month: 4 Day:15 (106/Year)
SSN: 3.3 IG: -7.5 (OBSERVED for the month)



foF2 (MHz) ~ LONGITUDE (Deg) and LATITUDE (Deg)

UT 18:00 Year:2008 Month: 4 Day:15 (106/Year)
SSN: 3.3 IG: -7.5 (OBSERVED for the month)



Adjusting the coefficients

$$\begin{aligned}
 foF2 = & \sum_{k=0}^K U_{0,0,k} \sin^k \chi + \sum_{j=1}^J \sum_{k=0}^K \left[U_{0,j,2k} \cos(j\varphi) \cos^j \lambda + U_{0,j,2k+1} \sin(j\varphi) \cos^j \lambda \right] \sin^k \chi \\
 & + \sum_{i=1}^I \sum_{k=0}^K \left[U_{2i-1,0,k} \sin(iT) + U_{2i,0,k} \cos(iT) \right] \sin^k \chi \\
 & + \sum_{i=1}^I \sum_{j=1}^J \sum_{k=0}^K \left[U_{2i-1,j,2k} \sin(iT) \cos(j\varphi) \cos^j \lambda + U_{2i,j,2k} \cos(iT) \cos(j\varphi) \cos^j \lambda \right. \\
 & \quad \left. + U_{2i-1,j,2k+1} \sin(iT) \sin(j\varphi) \cos^j \lambda + U_{2i,j,2k+1} \cos(iT) \sin(j\varphi) \cos^j \lambda \right] \sin^k \chi \\
 K = & \max \{ K(j), j = 0, J \}
 \end{aligned}$$

- The method to be developed:
 - The summation for (I, j, k) can be exchanged.
 - Basic idea: Adjust coefficients.
 - Using the mathematical tool: **Elastic Linear Optimization (ELO)**.

Extension of the index ranges

$$foF2 = \sum_{k=0}^{K(0)} \sin^k \chi \left[U_{0,0,k} + \sum_{i=1}^I (U_{2i-1,0,k} \sin(iT) + U_{2i,0,k} \cos(iT)) \right] \\ + \sum_{j=1}^J \sum_{k=0}^{K(j)} \left[\cos(j\phi) \cos^j \lambda \sin^k \chi \left[U_{0,j,2k} + \sum_{i=1}^I (U_{2i-1,j,2k} \sin(iT) + U_{2i,j,2k} \cos(iT)) \right] \right. \\ \left. + \sin(j\phi) \cos^j \lambda \sin^k \chi \left[U_{0,j,2k+1} + \sum_{i=1}^I (U_{2i-1,j,2k+1} \sin(iT) + U_{2i,j,2k+1} \cos(iT)) \right] \right]$$

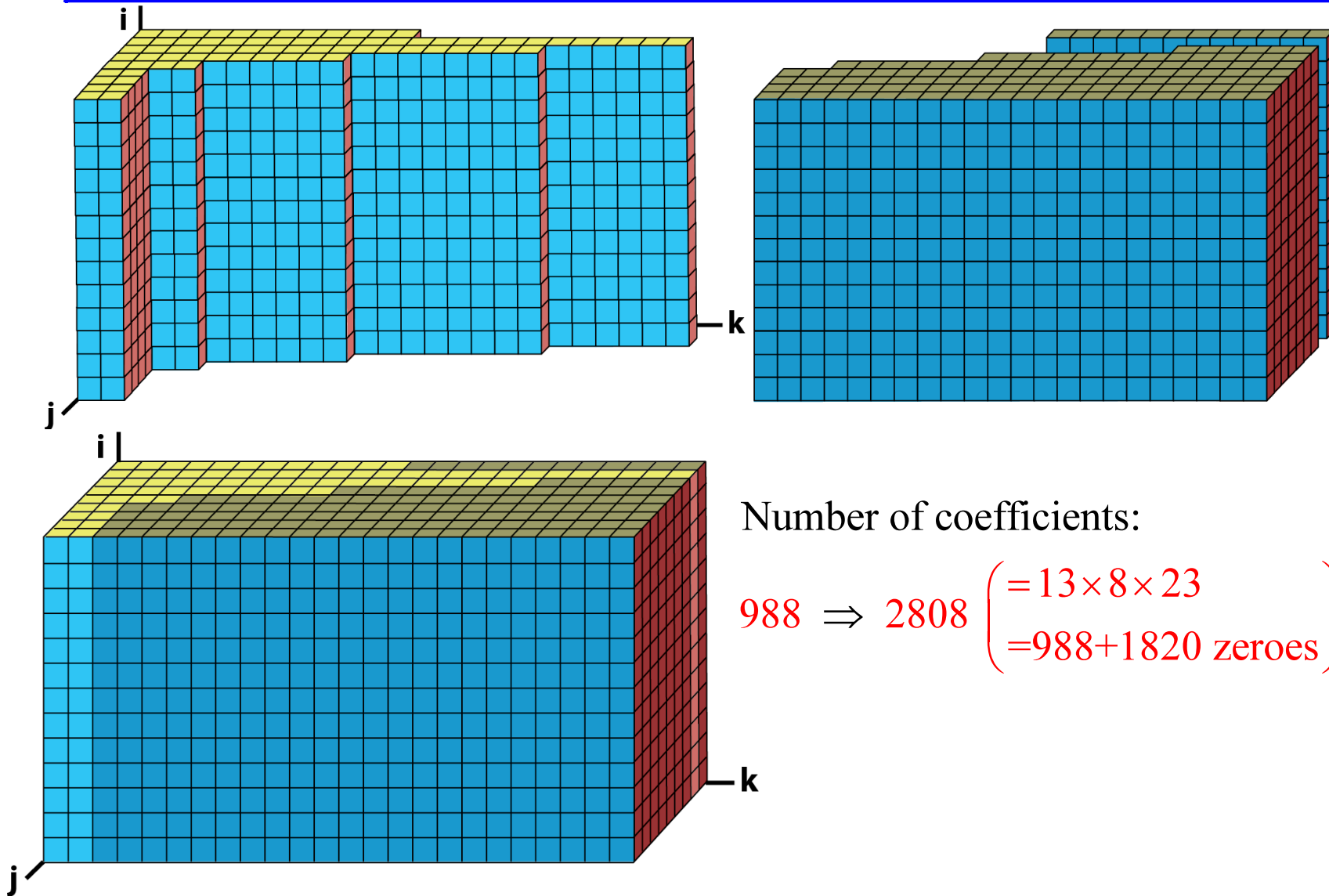
Extend the index k to a constant K , $K = \text{MAX} \{K(j), j \in [0, J]\}$ and fill zeroes in all the extended elements

$$\left\{ \begin{matrix} U_{2i-1,j,2k} = 0 \\ U_{2i-1,j,2k+1} = 0 \end{matrix} \right\} \text{ for } \left\{ \begin{matrix} i \in [1, I] \\ j \in [0, J] \\ k > K(j) \end{matrix} \right\}; \quad \left\{ \begin{matrix} U_{2i,j,2k} = 0 \\ U_{2i,j,2k+1} = 0 \end{matrix} \right\} \text{ for } \left\{ \begin{matrix} i \in [0, I] \\ j \in [0, J] \\ k > K(j) \end{matrix} \right\}$$

$$foF2 = \sum_{k=0}^K \sin^k \chi \left[U_{0,0,k} + \sum_{i=1}^I \left(U_{2i-1,0,k} \sin(iT) + U_{2i,0,k} \cos(iT) \right) \right] \\ + \sum_{j=1}^J \sum_{k=0}^K \left[U_{0,j,2k} \cos(j\phi) \cos^j \lambda \sin^k \chi + U_{0,j,2k+1} \sin(j\phi) \cos^j \lambda \sin^k \chi \right] \\ + \sum_{j=1}^J \sum_{k=0}^K \left[\cos(j\phi) \cos^j \lambda \sin^k \chi \sum_{i=1}^I \left(U_{2i-1,j,2k} \sin(iT) + U_{2i,j,2k} \cos(iT) \right) \right. \\ \left. + \sin(j\phi) \cos^j \lambda \sin^k \chi \sum_{i=1}^I \left(U_{2i-1,j,2k+1} \sin(iT) + U_{2i,j,2k+1} \cos(iT) \right) \right]$$

Data structure of the extended coefficients

$$foF2: U_{i,j,k}, \quad i \in [0,12], \quad j \in [0,8], \quad k \in [0,23]$$



Number of coefficients:

$$988 \Rightarrow 2808 \left(\begin{array}{l} = 13 \times 8 \times 23 \\ = 988 + 1820 \text{ zeroes} \end{array} \right)$$

Initial coefficients (1)

$$foF2 = a_0(\varphi, \lambda, \chi) + \sum_{i=1}^I [a_{2i-1}(\varphi, \lambda, \chi) \sin(iT) + a_{2i}(\varphi, \lambda, \chi) \cos(iT)]$$

$$a_{2i}(\varphi, \lambda, \chi) = \sum_{k=0}^K \sin^k \chi \left[U_{2i,0,k} + \sum_{j=1}^J (U_{2i,j,2k} \cos(j\varphi) + U_{2i,j,2k+1} \sin(j\varphi)) \cos^j \lambda \right], \quad i = (0, I)$$

$$a_{2i-1}(\varphi, \lambda, \chi) = \sum_{k=0}^K \sin^k \chi \left[U_{2i-1,0,k} + \sum_{j=1}^J (U_{2i-1,j,2k} \cos(j\varphi) + U_{2i-1,j,2k+1} \sin(j\varphi)) \cos^j \lambda \right], \quad i = (1, I)$$

- Introduce a factor $F = R_{eff}/100$ to get the effective sunspot R_{eff}

$$\begin{aligned} foF2 &= F \left\{ a_0^{(0)}(\varphi, \lambda, \chi) + \sum_{i=1}^I [a_{2i-1}^{(0)}(\varphi, \lambda, \chi) \sin(iT) + a_{2i}^{(0)}(\varphi, \lambda, \chi) \cos(iT)] \right\} \\ &\quad + (1 - F) \left\{ a_0^{(100)}(\varphi, \lambda, \chi) + \sum_{i=1}^I [a_{2i-1}^{(100)}(\varphi, \lambda, \chi) \sin(iT) + a_{2i}^{(100)}(\varphi, \lambda, \chi) \cos(iT)] \right\} \\ &= \left\{ a_0^{(100)}(\varphi, \lambda, \chi) + \sum_{i=1}^I [a_{2i-1}^{(100)}(\varphi, \lambda, \chi) \sin(iT) + a_{2i}^{(100)}(\varphi, \lambda, \chi) \cos(iT)] \right\} \\ &\quad + F \left\{ [a_0^{(0)}(\varphi, \lambda, \chi) - a_0^{(100)}(\varphi, \lambda, \chi)] + \sum_{i=1}^I \left[(a_{2i-1}^{(0)}(\varphi, \lambda, \chi) - a_{2i-1}^{(100)}(\varphi, \lambda, \chi)) \sin(iT) \right. \right. \\ &\quad \left. \left. + (a_{2i}^{(0)}(\varphi, \lambda, \chi) - a_{2i}^{(100)}(\varphi, \lambda, \chi)) \cos(iT) \right] \right\} \end{aligned}$$

$$a_{2i}^{(r)}(\varphi, \lambda, \chi) = \sum_{k=0}^K \sin^k \chi \left[U_{2i,0,k}^{(r)} + \sum_{j=1}^J (U_{2i,j,2k}^{(r)} \cos(j\varphi) + U_{2i,j,2k+1}^{(r)} \sin(j\varphi)) \cos^j \lambda \right], \quad i = (0, I), r = 0, 100$$

$$a_{2i-1}^{(r)}(\varphi, \lambda, \chi) = \sum_{k=0}^K \sin^k \chi \left[U_{2i-1,0,k}^{(r)} + \sum_{j=1}^J (U_{2i-1,j,2k}^{(r)} \cos(j\varphi) + U_{2i-1,j,2k+1}^{(r)} \sin(j\varphi)) \cos^j \lambda \right], \quad i = (1, I), r = 0, 100$$

Initial coefficients (2)

- Determine the factor by minimizing (least squares fitting)

$$\varepsilon = \sum_{m=1}^M \sum_{n=1}^N \left\{ \left[a_0^{(100)}(\varphi_m, \lambda_m, \chi_m) + \sum_{i=1}^I \left[a_{2i-1}^{(100)}(\varphi_m, \lambda_m, \chi_m) \sin(iT_{mn}) + a_{2i}^{(100)}(\varphi_m, \lambda_m, \chi_m) \cos(iT_{mn}) \right] \right]^2 \right. \\ \left. + F \left\{ \left[a_0^{(0)}(\varphi_m, \lambda_m, \chi_m) - a_0^{(100)}(\varphi_m, \lambda_m, \chi_m) \right] \right. \right. \\ \left. \left. + \sum_{i=1}^I \left[\left[a_{2i-1}^{(0)}(\varphi_m, \lambda_m, \chi_m) - a_{2i-1}^{(100)}(\varphi_m, \lambda_m, \chi_m) \right] \sin(iT_{mn}) \right] \right. \right. \\ \left. \left. + \left[a_{2i}^{(0)}(\varphi_m, \lambda_m, \chi_m) - a_{2i}^{(100)}(\varphi_m, \lambda_m, \chi_m) \right] \cos(iT_{mn}) \right] \right\} - Q_{mn} \right\}$$

$$C_{mn}^{(100)} = a_0^{(100)}(\varphi_m, \lambda_m, \chi_m) + \sum_{i=1}^I \left[a_{2i-1}^{(100)}(\varphi_m, \lambda_m, \chi_m) \sin(iT_{mn}) + a_{2i}^{(100)}(\varphi_m, \lambda_m, \chi_m) \cos(iT_{mn}) \right]$$

$$C_{mn}^{(0-100)} = \left[a_0^{(0)}(\varphi_m, \lambda_m, \chi_m) - a_0^{(100)}(\varphi_m, \lambda_m, \chi_m) \right] + \sum_{i=1}^I \left[\left[a_{2i-1}^{(0)}(\varphi_m, \lambda_m, \chi_m) - a_{2i-1}^{(100)}(\varphi_m, \lambda_m, \chi_m) \right] \sin(iT_{mn}) \right. \\ \left. + \left[a_{2i}^{(0)}(\varphi_m, \lambda_m, \chi_m) - a_{2i}^{(100)}(\varphi_m, \lambda_m, \chi_m) \right] \cos(iT_{mn}) \right]$$

$$\varepsilon = \sum_{m=1}^M \sum_{n=1}^N \left\{ C_{mn}^{(100)} + F C_{mn}^{(0-100)} - Q_{mn} \right\}^2$$

$$\frac{\partial \varepsilon}{\partial F} = 2 \sum_{m=1}^M \sum_{n=1}^N \left\{ C_{mn}^{(100)} + F C_{mn}^{(0-100)} - Q_{mn} \right\} C_{mn}^{(0-100)} = 0, \quad \sum_{m=1}^M \sum_{n=1}^N \left\{ F C_{mn}^{(0-100)} \right\} C_{mn}^{(0-100)} = \sum_{m=1}^M \sum_{n=1}^N \left\{ C_{mn}^{(100)} - Q_{mn} \right\} C_{mn}^{(0-100)}$$

$$F = \frac{\sum_{m=1}^M \sum_{n=1}^N \left\{ C_{mn}^{(100)} - Q_{mn} \right\} C_{mn}^{(0-100)}}{\sum_{m=1}^M \sum_{n=1}^N C_{mn}^{(0-100)} C_{mn}^{(0-100)}}$$

Initial coefficients (3)

- The initial coefficients

$$U_{i,j,k} = U_{i,j,k}^{(100)} + \textcolor{red}{F} \left(U_{i,j,k}^{(0)} - U_{i,j,k}^{(100)} \right),$$
$$i \in [0, 2I], \quad j \in [0, J], \quad k \in [0, 2K + 1]$$
$$R_{eff} = 100 \textcolor{red}{F}$$

Adjustment for diurnal variation

(1) Diurnal variation

$$foF2 = a_0(\varphi, \lambda, \chi) + \sum_{i=1}^I [a_{2i-1}(\varphi, \lambda, \chi) \sin(iT) + a_{2i}(\varphi, \lambda, \chi) \cos(iT)]$$

$$a_{2i}(\varphi, \lambda, \chi) = \sum_{k=0}^K \sin^k \chi \left[U_{2i,0,k} + \sum_{j=1}^J (U_{2i,j,2k} \cos(j\varphi) + U_{2i,j,2k+1} \sin(j\varphi)) \cos^j \lambda \right], \quad i = (0, I)$$

$$a_{2i-1}(\varphi, \lambda, \chi) = \sum_{k=0}^K \sin^k \chi \left[U_{2i-1,0,k} + \sum_{j=1}^J (U_{2i-1,j,2k} \cos(j\varphi) + U_{2i-1,j,2k+1} \sin(j\varphi)) \cos^j \lambda \right], \quad i = (1, I)$$

- Introduce (2I+1=13) correction factors. Each factor adjusts one component.

$$F_i^{(T)}, i \in [0, 2I]$$

$$foF2 = F_0^{(T)} a_0(\varphi, \lambda, \chi) + \sum_{i=1}^I [F_{2i-1}^{(T)} a_{2i-1}(\varphi, \lambda, \chi) \sin(iT) + F_{2i}^{(T)} a_{2i}(\varphi, \lambda, \chi) \cos(iT)]$$

- Determine the factors by minimizing (least squares fitting)

$$\mathcal{E}^{(T)} = \sum_{m=1}^M \sum_{n=1}^N \left\{ F_0^{(T)} a_0(\varphi_m, \lambda_m, \chi_m) + \sum_{i=1}^I [F_{2i-1}^{(T)} a_{2i-1}(\varphi_m, \lambda_m, \chi_m) \sin(iT_{mn}) + F_{2i}^{(T)} a_{2i}(\varphi_m, \lambda_m, \chi_m) \cos(iT_{mn})] - Q_{mn} \right\}^2$$

- Adjust the coefficients

$$F_i^{(T)} U_{i,j,k} \Rightarrow U_{i,j,k}, \quad i \in [0, 2I], \quad j \in [0, J], \quad k \in [0, 2K+1]$$

Adjustment for geographic location variation

(2) Location variation

$$foF2 = b_0(T, \chi) + \sum_{j=1}^J \left[b_{2j-1}(T, \chi) \sin(j\varphi) \cos^j \lambda + b_{2j}(T, \chi) \cos(j\varphi) \cos^j \lambda \right]$$

$$b_0(T, \chi) = \sum_{k=0}^K \sin^k \chi \left[U_{0,0,k} + \sum_{i=1}^I (U_{2i-1,0,k} \sin(iT) + U_{2i,0,k} \cos(iT)) \right]$$

$$b_{2j}(T, \chi) = \sum_{k=0}^K \sin^k \chi \left[U_{0,j,2k} + \sum_{i=1}^I (U_{2i-1,j,2k} \sin(iT) + U_{2i,j,2k} \cos(iT)) \right], \quad j \in [1, J]$$

$$b_{2j-1}(T, \chi) = \sum_{k=0}^K \sin^k \chi \left[U_{0,j,2k+1} + \sum_{i=1}^I (U_{2i-1,j,2k+1} \sin(iT) + U_{2i,j,2k+1} \cos(iT)) \right], \quad j \in [1, J]$$

- Introduce (2J+1=17) correction factors

$$F_j^{(L)}, j \in [0, 2J]$$

$$foF2 = F_0^{(L)} b_0(T, \chi) + \sum_{j=1}^J \left[F_{2j-1}^{(L)} b_{2j-1}(T, \chi) \sin(j\varphi) \cos^j \lambda + F_{2j}^{(L)} b_{2j}(T, \chi) \cos(j\varphi) \cos^j \lambda \right]$$

- Determine the factors by minimizing

$$\varepsilon^{(L)} = \sum_{m=1}^M \sum_{n=1}^N \left\{ F_0^{(L)} b_0(T_{mn}, \chi_m) + \sum_{j=1}^J \left[F_{2j-1}^{(L)} b_{2j-1}(T_{mn}, \chi_m) \sin(j\varphi_m) \cos^j \lambda_m \right] + F_{2j}^{(L)} b_{2j}(T_{mn}, \chi_m) \cos(j\varphi_m) \cos^j \lambda_m \right\}^2 - Q_{mn}$$

- Adjust the coefficients

$$\left\{ F_0^{(L)} U_{i,0,k} \Rightarrow U_{i,0,k} \right\}, \quad i \in [0, 2I], k \in [0, K]$$

$$\left\{ \begin{array}{l} F_{2j}^{(L)} U_{i,j,2k} \Rightarrow U_{i,j,2k} \\ F_{2j-1}^{(L)} U_{i,j,2k+1} \Rightarrow U_{i,j,2k+1} \end{array} \right\}, \quad i \in [0, 2I], j \in [1, J], k \in [0, K]$$

Adjustment for geomagnetic location variation

(3) Geomagnetic-latitudal variation

$$foF2 = \sum_{k=0}^K c_k(T, \varphi, \lambda) \sin^k \chi$$

$$c_k(T, \varphi, \lambda) = \left\{ \begin{aligned} &U_{0,0,k} + \sum_{i=1}^I (U_{2i-1,0,k} \sin(iT) + U_{2i,0,k} \cos(iT)) \\ &+ \sum_{j=1}^J \cos(j\varphi) \cos^j \lambda \left[U_{0,j,2k} + \sum_{i=1}^I (U_{2i-1,j,2k} \sin(iT) + U_{2i,j,2k} \cos(iT)) \right] \\ &+ \sum_{j=1}^J \sin(j\varphi) \cos^j \lambda \left[U_{0,j,2k+1} + \sum_{i=1}^I (U_{2i-1,j,2k+1} \sin(iT) + U_{2i,j,2k+1} \cos(iT)) \right] \end{aligned} \right\}, \quad k \in [0, K]$$

- Introduce ($K+1=12$) correction factors.

$$F_k^{(M)}, \quad k \in [0, K]$$

$$foF2 = \sum_{k=0}^K F_k^{(M)} c_k(T, \varphi, \lambda) \sin^k \chi$$

- Determine the factors by minimizing

$$\mathcal{E}^{(M)} = \sum_{m=1}^M \sum_{n=1}^N \left\{ \sum_{k=0}^K F_k^{(M)} c_k(T_{mn}, \varphi_m, \lambda_m) \sin^k \chi_m - Q_{mn} \right\}^2$$

- Adjust the coefficients

$$\left\{ \begin{aligned} &F_k^{(M)} U_{i,j,2k} \Rightarrow U_{i,j,2k} \\ &F_k^{(M)} U_{i,j,2k+1} \Rightarrow U_{i,j,2k+1} \end{aligned} \right\}, \quad i \in [0, 2I], \quad j \in [0, J], \quad k \in [0, K]$$

Adjustment for geomagnetic correction

(3) Geomagnetic-latitudal
correction

$$foF2 = \sum_{k=0}^K c_k^{(0)}(T, \varphi, \lambda) \sin^k \chi + \sum_{k=0}^K c_k^{(2)}(T, \varphi, \lambda) \sin^k \chi + \sum_{k=0}^K c_k^{(1)}(T, \varphi, \lambda) \sin^k \chi$$

$$c_k^{(0)}(T, \varphi, \lambda) = U_{0,0,k} + \sum_{i=1}^I (U_{2i-1,0,k} \sin(iT) + U_{2i,0,k} \cos(iT)), \quad k \in [0, K]$$

$$c_k^{(2)}(T, \varphi, \lambda) = \sum_{j=1}^J \cos(j\varphi) \cos^j \lambda \left[U_{0,j,2k} + \sum_{i=1}^I (U_{2i-1,j,2k} \sin(iT) + U_{2i,j,2k} \cos(iT)) \right], \quad k \in [0, K]$$

$$c_k^{(1)}(T, \varphi, \lambda) = \sum_{j=1}^J \sin(j\varphi) \cos^j \lambda \left[U_{0,j,2k+1} + \sum_{i=1}^I (U_{2i-1,j,2k+1} \sin(iT) + U_{2i,j,2k+1} \cos(iT)) \right], \quad k \in [0, K]$$

- Introduce $(3(K+1)=36)$ correction factors.

$$F_k^{(M,0)}, F_k^{(M,2)}, F_k^{(M,1)}, \quad k \in [0, K]$$

$$foF2 = \sum_{k=0}^K \left[F_k^{(M,0)} c_k^{(0)}(T_{mn}, \varphi_m, \lambda_m) + F_k^{(M,2)} c_k^{(2)}(T_{mn}, \varphi_m, \lambda_m) + F_k^{(M,1)} c_k^{(1)}(T_{mn}, \varphi_m, \lambda_m) \right] \sin^k \chi$$

- Determine the factors by minimizing

$$\varepsilon^{(M)} = \sum_{m=1}^M \sum_{n=1}^N \left\{ \sum_{k=0}^K \left[F_k^{(M,0)} c_k^{(0)}(T_{mn}, \varphi_m, \lambda_m) + F_k^{(M,2)} c_k^{(2)}(T_{mn}, \varphi_m, \lambda_m) + F_k^{(M,1)} c_k^{(1)}(T_{mn}, \varphi_m, \lambda_m) \right] \sin^k \chi_m - Q_{mn} \right\}^2$$

- Adjust the coefficients

$$\left\{ \begin{array}{l} F_k^{(M,0)} U_{i,0,k} \Rightarrow U_{i,0,k}, \quad i \in [0, 2I], \quad j = 0, \quad k \in [0, K] \\ F_k^{(M,2)} U_{i,j,2k} \Rightarrow U_{i,j,2k}, \quad i \in [0, 2I], \quad j \in [1, J], \quad k \in [0, K] \\ F_k^{(M,1)} U_{i,j,2k+1} \Rightarrow U_{i,j,2k+1}, \quad i \in [0, 2I], \quad j \in [1, J], \quad k \in [0, K] \end{array} \right\}$$

Real time data

Real time data (from GIRO or any other data base)

Station location: (φ_m, λ_m) , $m = 1, 2, \dots, M$

Observations in the last 24 hours: T_{mn} , $n = 1, 2, \dots, N$

(the data samples may not be evenly spaced):

$$\{Q_{mn} \equiv Q(\varphi_m, \lambda_m; T_{mn}), \quad m = 1, 2, \dots, M; \quad n = 1, 2, \dots, N\}$$

Data quality: Most data points are reliable

If $M = 40$ and $N = 4 \times 24 = 96$,

Total data points = $M \times N = 3840$

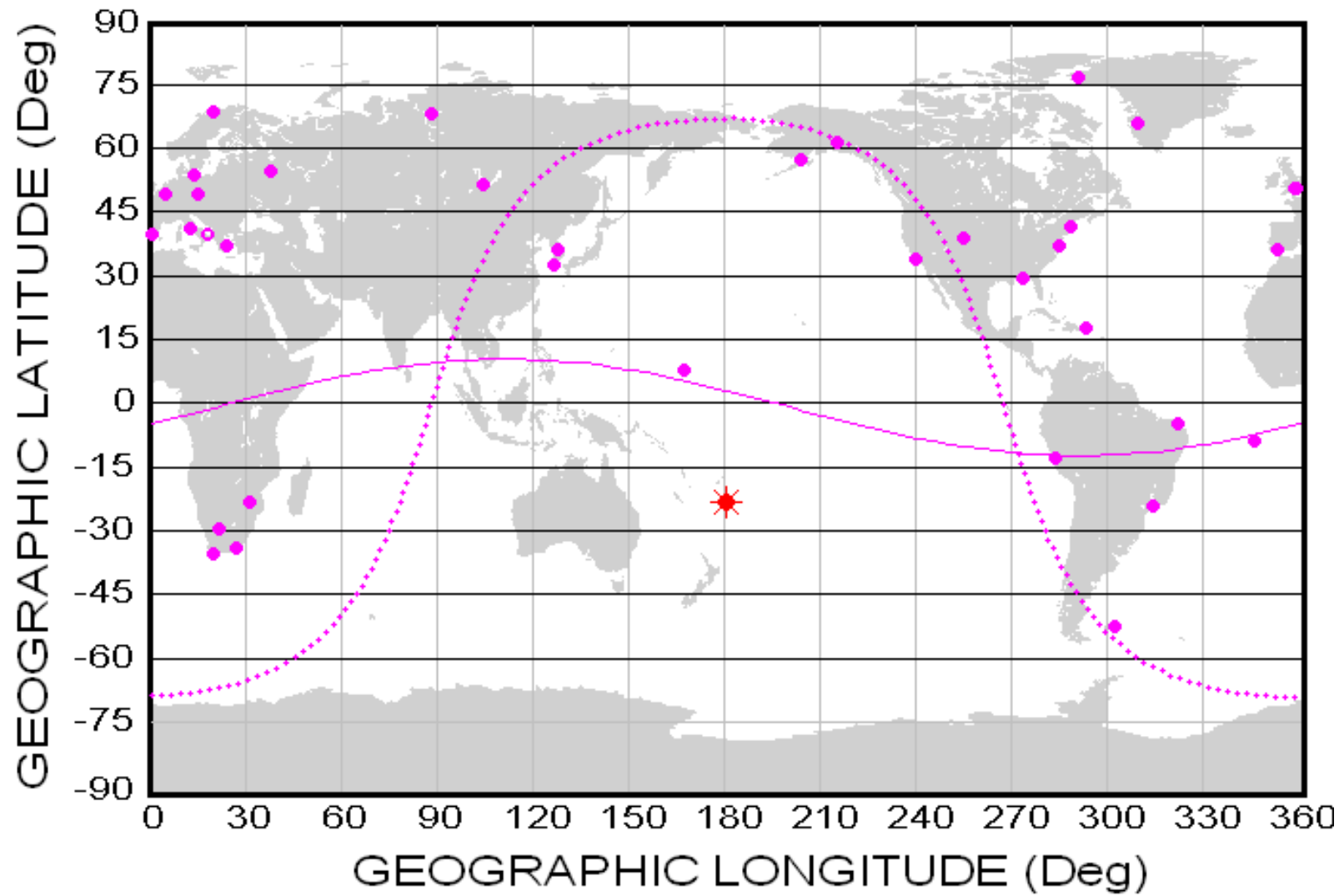
Considering the possibility of missing data,

the expected data points (tests) = $\langle 2500, 3000 \rangle$

Preliminary test results

- Data:
 - 2011 December.
 - From 36 stations
- ELO Method
 - For a given UT time t , all the data for the previous 24 hours data are used.
 - To adjust the coefficients so that for the three variations: Diurnal variation, Geographic location variation and magnetic location variations are best match with the measured data.
 - In this method, each coefficient is adjusted but avoid to deal with large matrix equations.

GIRO Stations



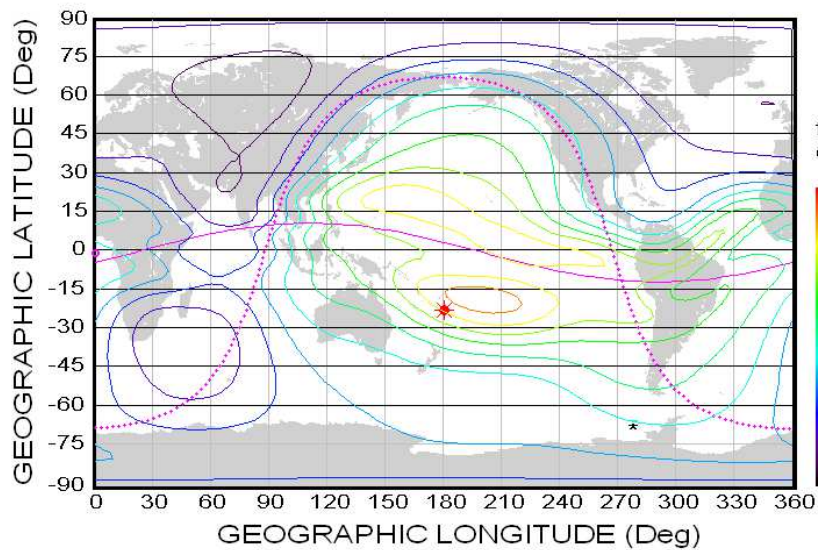
Testing Results – Global Map

00 UT, December 2, 2011

foF2 (MHz) ~ LONGITUDE (Deg) and LATITUDE (Deg)

UT 00:00 Year:2011 Month:12 Day: 2 (336/Year)

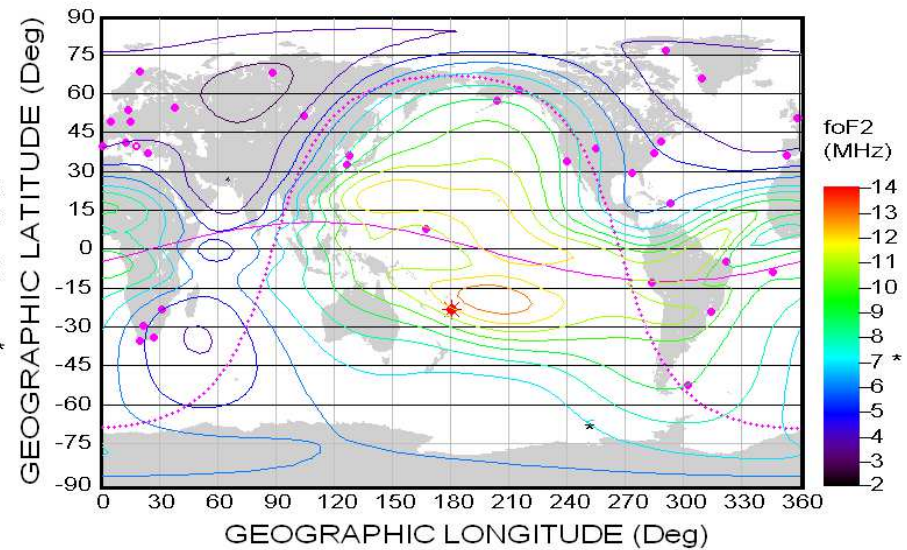
SSN: 61.0+/-16.6 IG: 67.3+/- 0.0 (PREDICTED for the month)



foF2 (MHz) ~ LONGITUDE (Deg) and LATITUDE (Deg)

UT 00:00 Year:2011 Month:12 Day: 2 (336/Year)

SSN: 61.0+/-16.6 IG: 67.3+/- 0.0 (PREDICTED for the month)



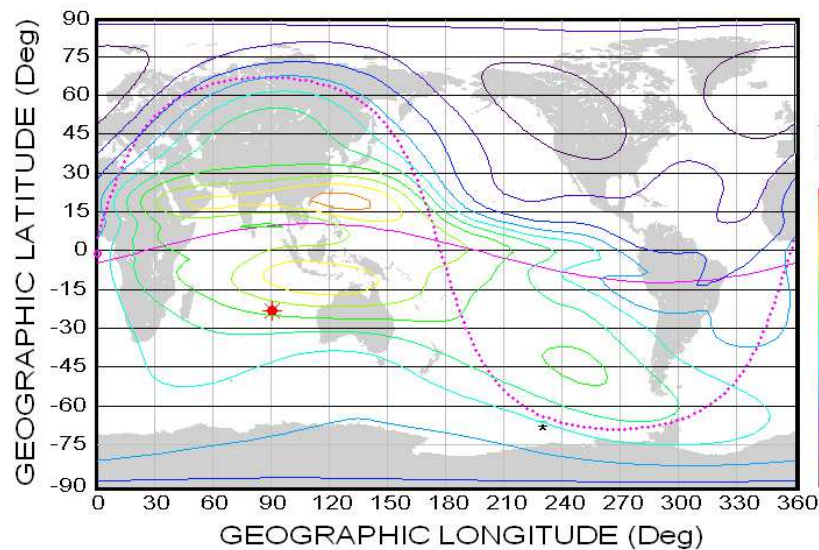
Testing Results – Global Map

06 UT, December 2, 2011

foF2 (MHz) ~ LONGITUDE (Deg) and LATITUDE (Deg)

UT 06:00 Year:2011 Month:12 Day: 2 (336/Year)

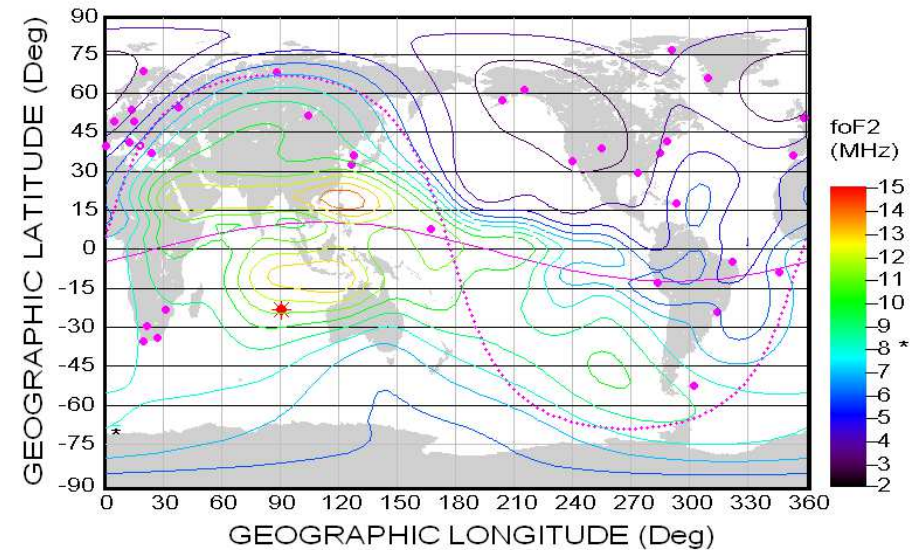
SSN: 61.0+/-16.6 IG: 67.3+/- 0.0 (PREDICTED for the month)



foF2 (MHz) ~ LONGITUDE (Deg) and LATITUDE (Deg)

UT 06:00 Year:2011 Month:12 Day: 2 (336/Year)

SSN: 61.0+/-16.6 IG: 67.3+/- 0.0 (PREDICTED for the month)

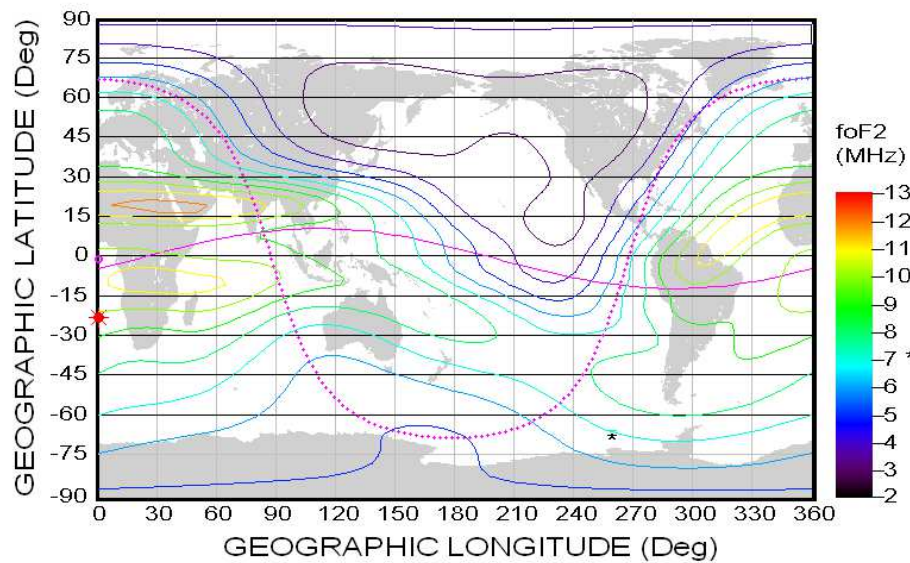


Testing Results – Global Map

12 UT, December 2, 2011

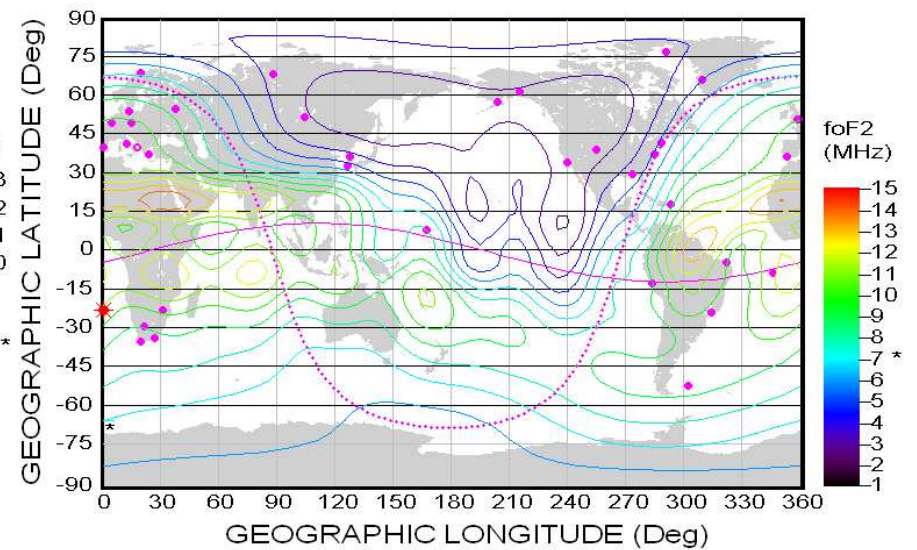
foF2 (MHz) ~ LONGITUDE (Deg) and LATITUDE (Deg)

UT 12:00 Year:2011 Month:12 Day: 2 (336/Year)
SSN: 61.0 \pm 16.6 IG: 67.3 \pm 0.0 (PREDICTED for the month)



foF2 (MHz) ~ LONGITUDE (Deg) and LATITUDE (Deg)

UT 12:00 Year:2011 Month:12 Day: 2 (336/Year)
SSN: 61.0 \pm 16.6 IG: 67.3 \pm 0.0 (PREDICTED for the month)



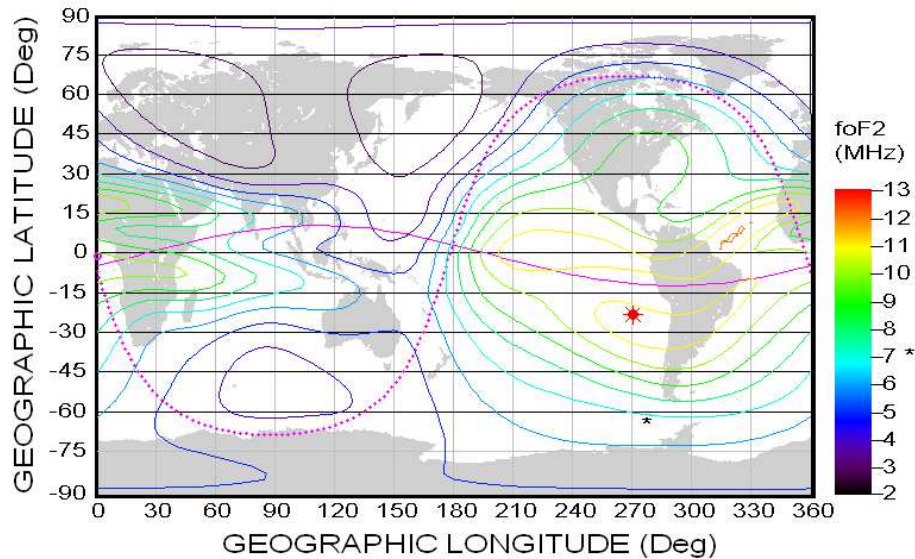
Testing Results – Global Map

18 UT, December 2, 2011

foF2 (MHz) ~ LONGITUDE (Deg) and LATITUDE (Deg)

UT 18:00 Year:2011 Month:12 Day: 2 (336/Year)

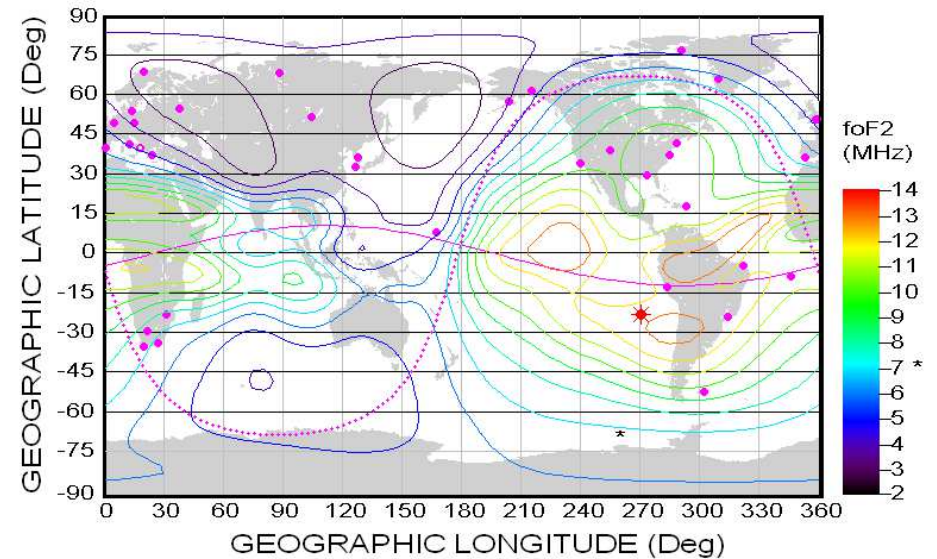
SSN: 61.0+/-16.6 IG: 67.3+/- 0.0 (PREDICTED for the month)



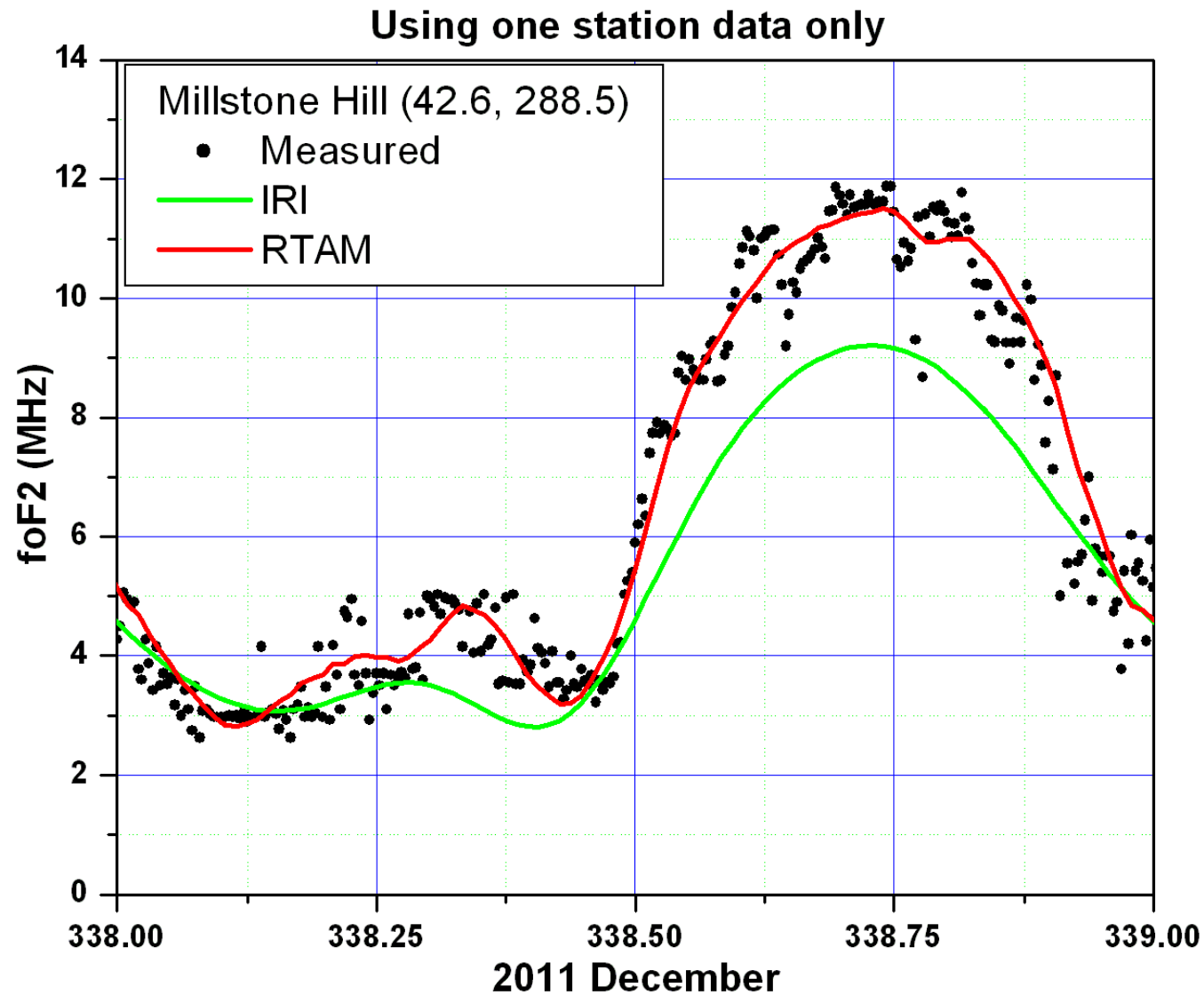
foF2 (MHz) ~ LONGITUDE (Deg) and LATITUDE (Deg)

UT 18:00 Year:2011 Month:12 Day: 2 (336/Year)

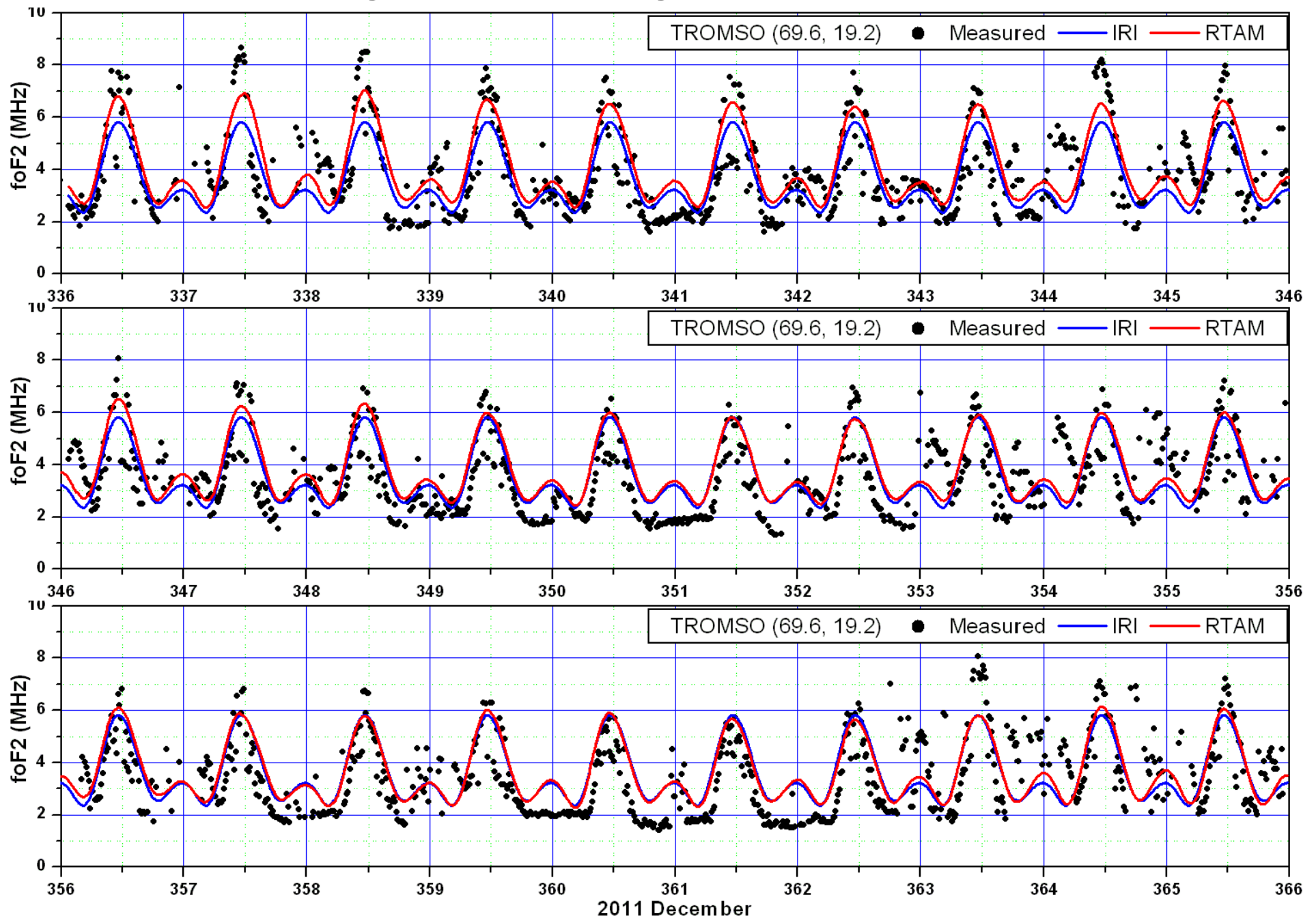
SSN: 61.0+/-16.6 IG: 67.3+/- 0.0 (PREDICTED for the month)



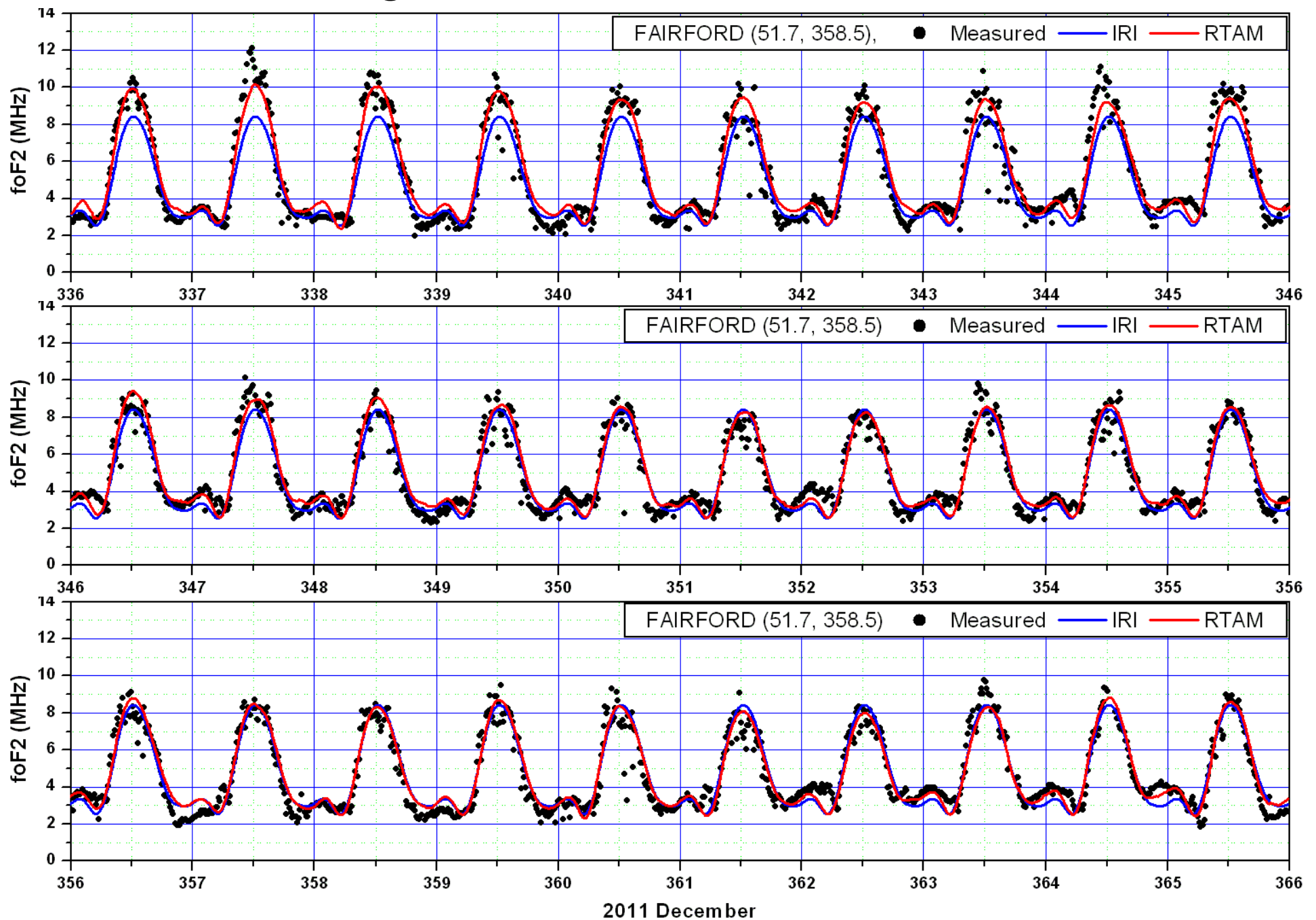
Testing Results – Using one station data only



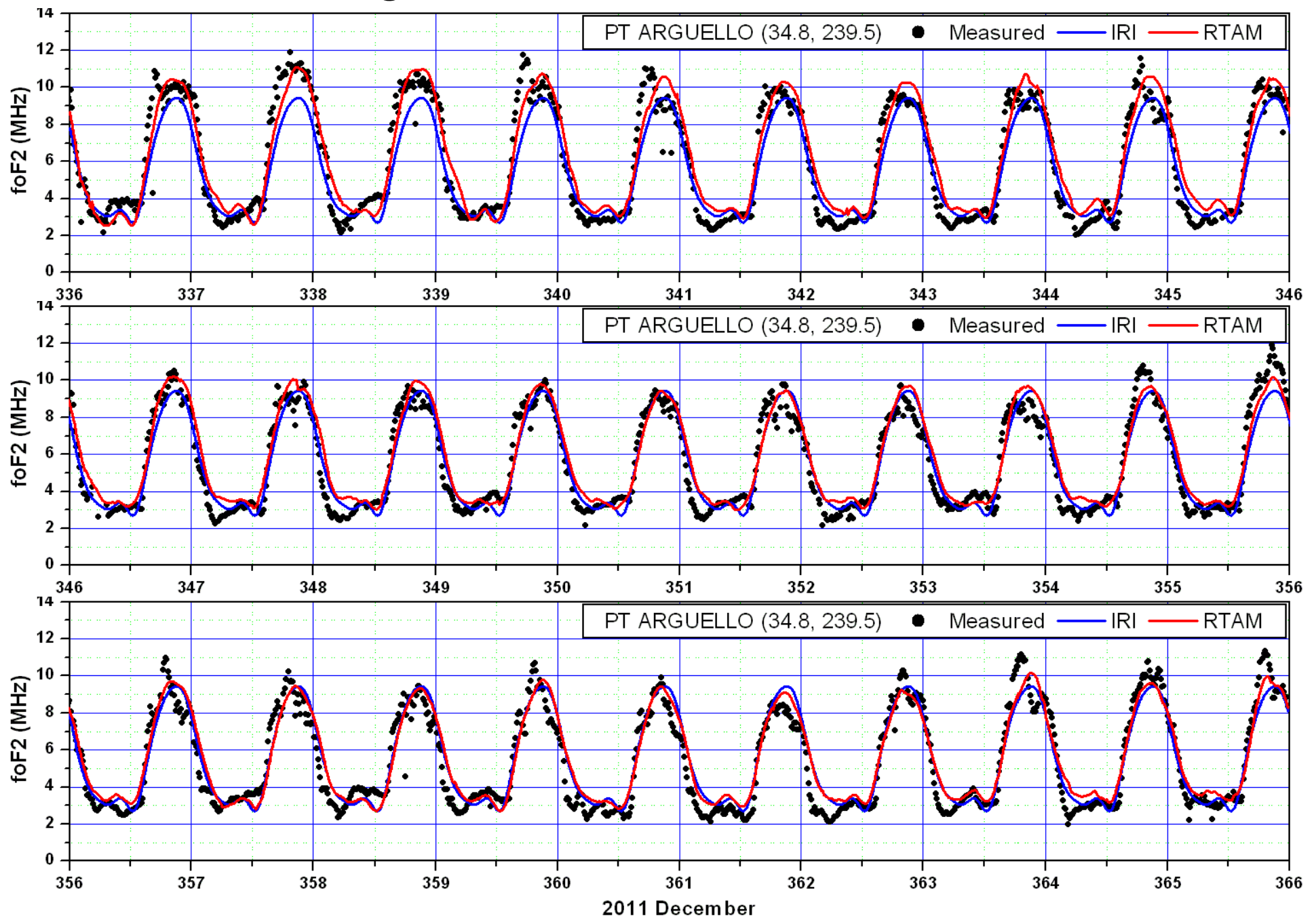
Testing Results – High Latitude Station



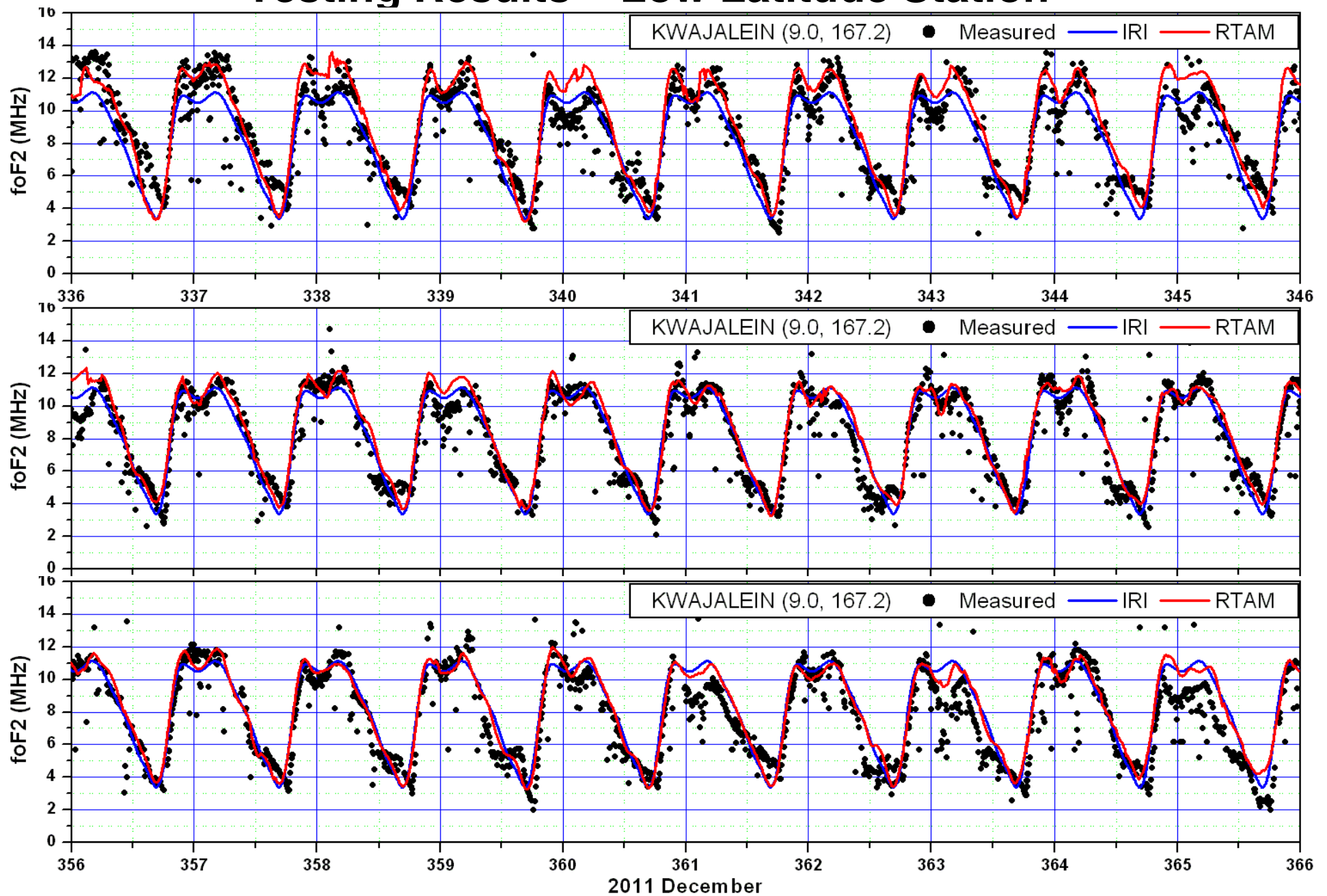
Testing Results – Mid Latitude Station



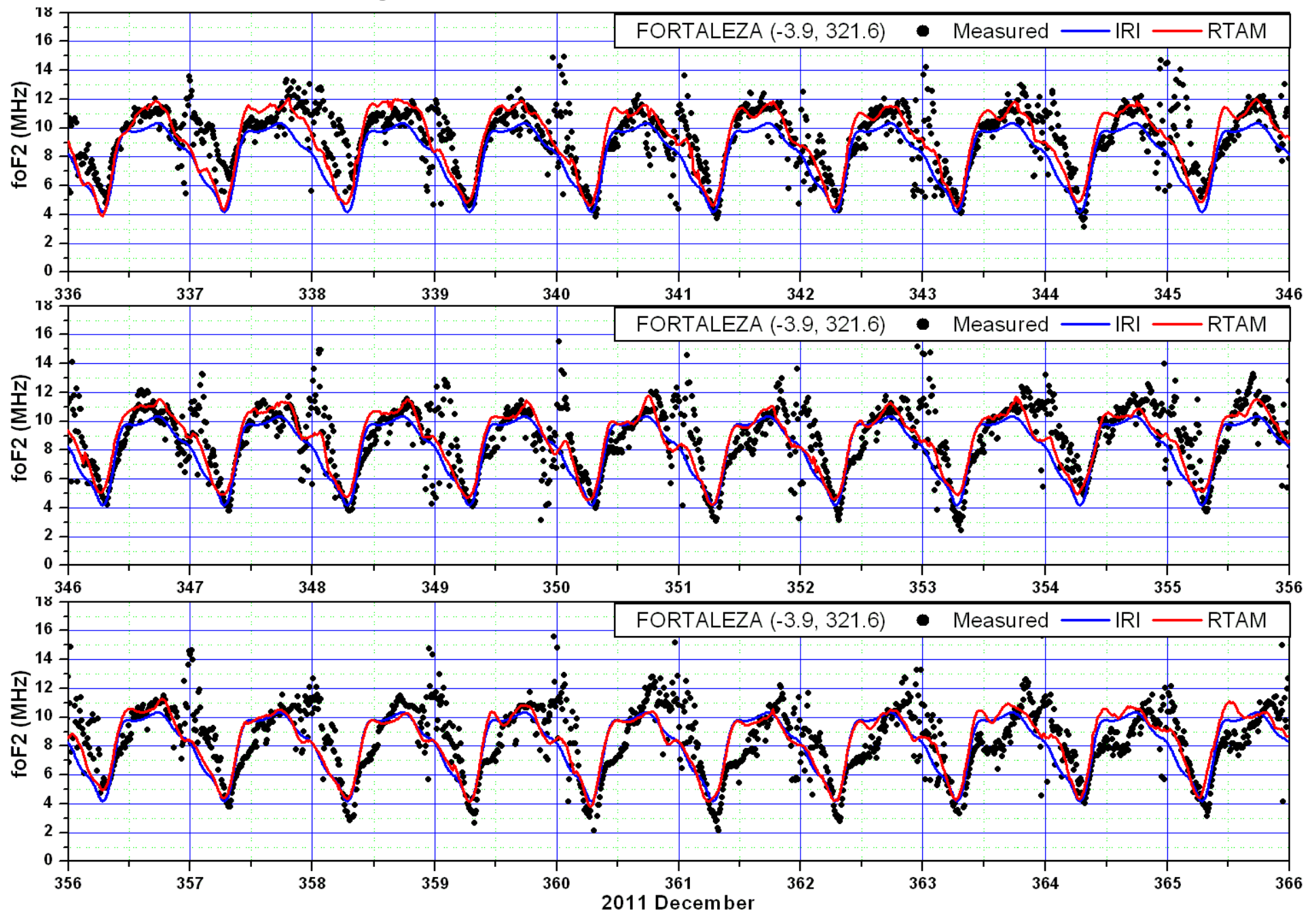
Testing Results – Mid Latitude Station



Testing Results – Low Latitude Station



Testing Results – Low Latitude Station



Discussions on ELO

- The adjustment for variations is to check and adjust the coefficients as to best match the observed temporal or spatial dependences of all the variation components. A group of correction factors adjusts not a few but the whole set of coefficients.
- There are no troubles arising from missing data point and/or using a few non-reliable data as the process does not directly check the components individually.
- The method is transparent and it can be applied to a small area using data from a single station. The input data is not limited to the data from the ground-based ionosonde network. Especially, mapping results can be obtained by this method using satellite measurements such as those from several topside sounding satellites.
- As it is believed that the model is good enough to describe the reality, the adjusted model should be valid to give a good estimation in an area with measured data. (In the theory of linear optimization, it has been proved that the adjusted result is converging for an increasing number of tests and independent of the initial model).

ELO is Applied to GIRO (EXAMPLE)